Sample Size Determination (Statistical Tolerance Limits)

Summary

This procedure determines a suitable sample size for estimating statistical tolerance limits. Statistical tolerance limits give a range of values for X such that one may be $100(1-\alpha)\%$ confident that *P* percent of the population from which a data sample comes falls within that range. This procedure determines sample sizes for several types of limits:

1. *Normal tolerance limits*: these limits assume that the data are a random sample from a normal distribution.

2. *Lognormal tolerance limits*: these limits assume that the data are a random sample from a lognormal distribution.

3. *Weibull tolerance limits*: these limits assume that the data are a random sample from a Weibull distribution.

4. *Nonparametric limits*: these limits do not assume that the data come from any particular distribution.

Sample StatFolio: *tolsize.sgp*

Sample Data:
None.

Data Input

The first dialog box displayed by this procedure is used to specify the problem of interest to the analyst.

- **Distribution:** the assumed distribution on which the tolerance limits will be based (if any). If a specific distribution is selected, the value of the parameters for that distribution must also be specified. In the case of the lognormal and Weibull distributions, which usually begin at X = 0, a non-zero threshold may also be specified.

- **Type of Limits** – either a two-sided tolerance interval or a one-sided tolerance bound may be requested.

- **Confidence Level** – the confidence level at which the limits will be calculated.

- **Population Proportion** – the proportion of the population bound by the tolerance limits.

- **Confidence Level**: the level of confidence for the limits.

- **Lower Spec. Limit**: the lower specification limit, if any. For bounds based on a specified distribution, a sample size *n* will be selected such that the lower tolerance limit is at or above this value.

- **Upper Spec. Limit**: the upper specification limit, if any. For bounds based on a specified distribution, a sample size *n* will be selected such that the upper tolerance limit is at or below this value.

- **Allowance**: the percentage of the distance between the mean and the specification limits that the tolerance limits may encompass. An allowance of 100% permits the tolerance limits to use the entire distance between the specs. A smaller allowance keeps the tolerance limits within the specs.

For example, the above dialog box requests a sample size for calculating 95% tolerance limits for 99% of the items in a population that follows a normal distribution with a mean equal to 50 and a standard deviation equal to 7. The tolerance interval may be no wider than 75% of the distance between the specifications limits, which range from 20 to 80.

Analysis Summary

The *Analysis Summary* displays the required sample size:

Sample Size Determination (Statistical Tolerance Intervals)

Conf. Level	Pop. Percentage	Distribution	Mean	Std. dev.	Lower spec.	Upper spec.	Allowance
95.0%	99.0%	Normal	50.0	7.0	20.0	80.0	75.0%

The required sample size is 41.

Lower tolerance limit	Upper tolerance limit	K-factor
27.5464	72.4536	3.20765

The StatAdvisor
Given a sample from a normal distribution with a mean equal to 50.0 and a standard deviation equal to 7.0, 41 items are required to create a 95.0% tolerance interval for 99.0% of the population which is entirely within the specification limits.


In the current example, a sample of $n = 41$ observations is required to achieve the desired tolerance limits.

NOTES:

1. It is not always possible to achieve the desired limits, regardless of how large the sample is. In such cases, an error message will be displayed.

2. This procedure determines a sample size by controlling the location and width of the tolerance limits when the sample mean and sample standard deviation equal the specified distributional parameters. An allowance of less than 100% may be desirable to compensate for natural variability in the sample statistics.

Tolerance Plot

The *Tolerance Plot* shows the assumed distribution together with the tolerance limits:

Normal Distribution
mean=50.0, std. dev.=7.0
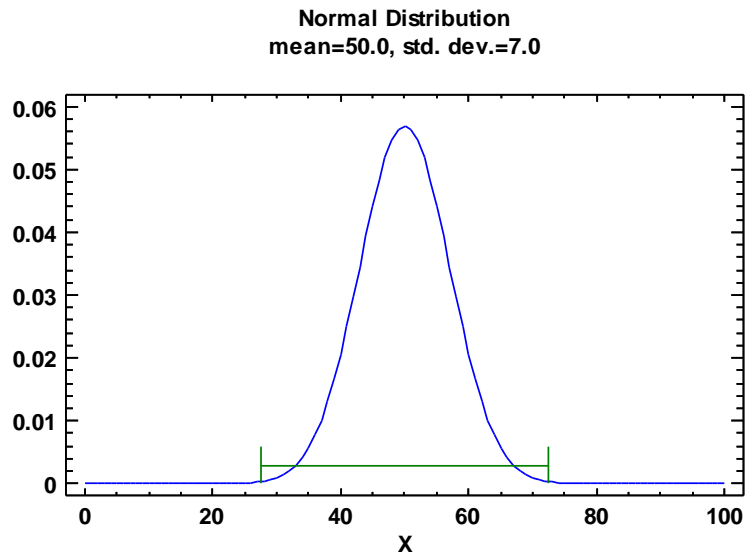
The limits are drawn assuming that $\bar{x} = \mu$ and $s = \sigma$.

Calculations

Let

LSL = lower specification limit (if any)

USL = upper specification limit (if any)

λ = allowance as a fraction

Normal Distribution

In the case of a normal distribution, tolerance limits are calculated from one of the following equations:

2-sided interval:

$$\bar{x} \pm Ks \tag{1}$$

1-sided bound:

$$\bar{x} - Ks \quad \text{or} \quad \bar{x} + Ks \tag{2}$$

where the factor K depends on the sample size, the confidence level, and the population proportion. Letting μ be the specified mean and σ be the specified standard deviation, STATGRAPHICS finds the smallest n such that

$$K\sigma \leq \lambda(\mu - LSL) \tag{3}$$

and

$$K\sigma \leq \lambda(USL - \mu) \tag{4}$$

ignoring the upper or lower limit if not requested.

Lognormal Distribution

In the case of a lognormal distribution, the entire problem is first transformed to the log scale and the methodology for the normal distribution is applied to the logs.

Weibull Distribution

In the case of a Weibull distribution, the specified Weibull distribution and specification limits are first transformed to a Weibull distribution with $\alpha = 4$ and $\beta = 100$. A normal approximation for that distribution is then calculated, and the methodology for the normal distribution is applied.

Nonparametric Limits

Nonparametric tolerance limits are based on the smallest and largest observed values in a data sample. STATGRAPHICS calculates the smallest n that will bound at least the desired population percentage at the specified confidence level.