GLM Method (Gage Studies – Variables)

statgraphics® centurion

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Summary

The *GLM Method* estimates the repeatability and reproducibility of a measurement system based on a study in which *m* appraisers measure *n* items *r* times. It also estimates important quantities such as the total variation, the precision-to-tolerance ratio, the standard deviation of the measurement error, and the percent of study contribution from various error components. In addition to variation introduced by appraisers and parts, additional factors may also be included. The additional factors may be treated as having either fixed or random effects.

Sample StatFolio: *gageglm.sgp*

Sample Data

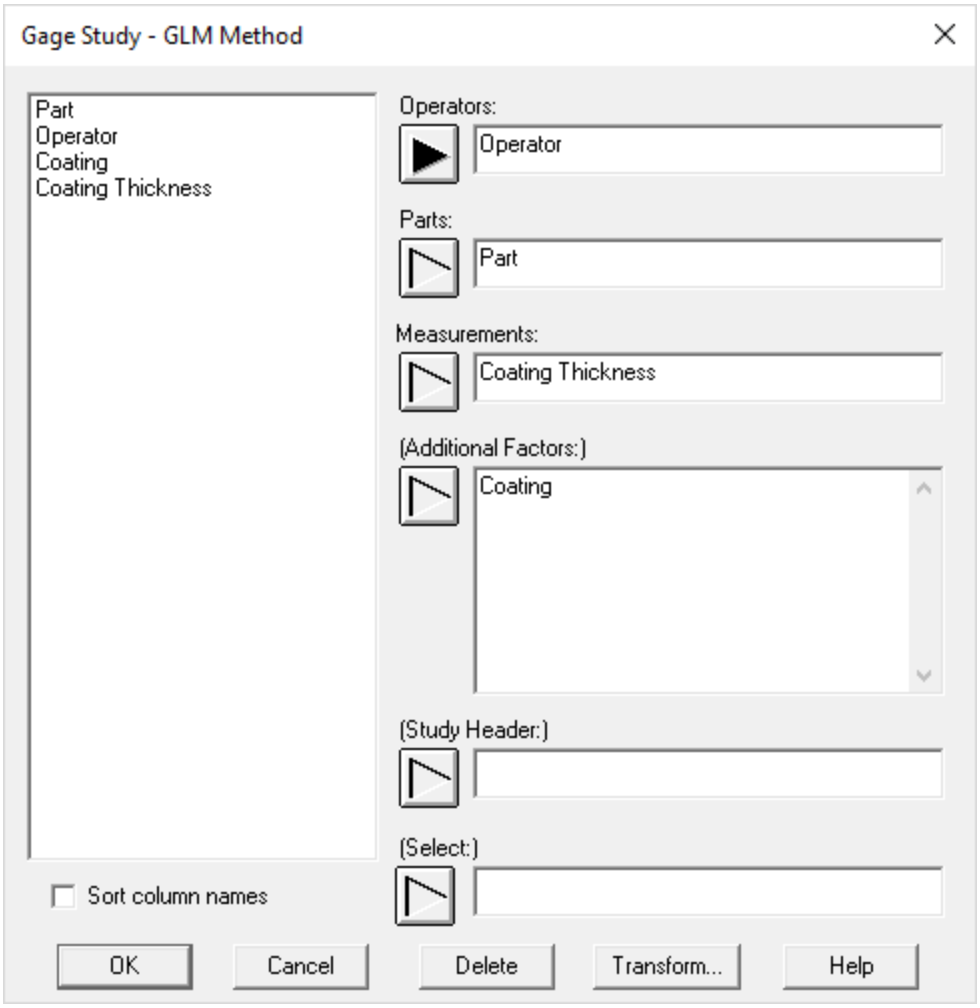
The file *coatings.sgd* contains data from a typical variables gage study. A partial list of the data in that file is shown below:

Part	*Operator*	*Coating*	*Coating Thickness*
1	Paul	Type A	34.47
1	Paul	Type A	36.25
1	Paul	Type B	39.28
1	Paul	Type B	38.80
2	Paul	Type A	21.20
2	Paul	Type B	23.95
2	Paul	Type A	21.71
2	Paul	Type B	24.07
3	Paul	Type B	49.40
3	Paul	Type A	47.08
3	Paul	Type A	48.91
3	Paul	Type B	48.92
4	Paul	Type A	37.77
4	Paul	Type A	37.07
4	Paul	Type B	40.69
4	Paul	Type B	39.99
5	Paul	Type A	19.39
5	Paul	Type B	21.31
5	Paul	Type A	17.41
5	Paul	Type B	20.37
6	Paul	Type A	30.52
6	Paul	Type A	28.14

The file contains a total of 120 rows, one for each of $r = 4$ measurements made by each of $m = 3$ operators on $n = 10$ parts. Of the 4 measurements made on each part, 2 involved applying a Type A coating while the other 2 involved applying a Type B coating.

Data Input

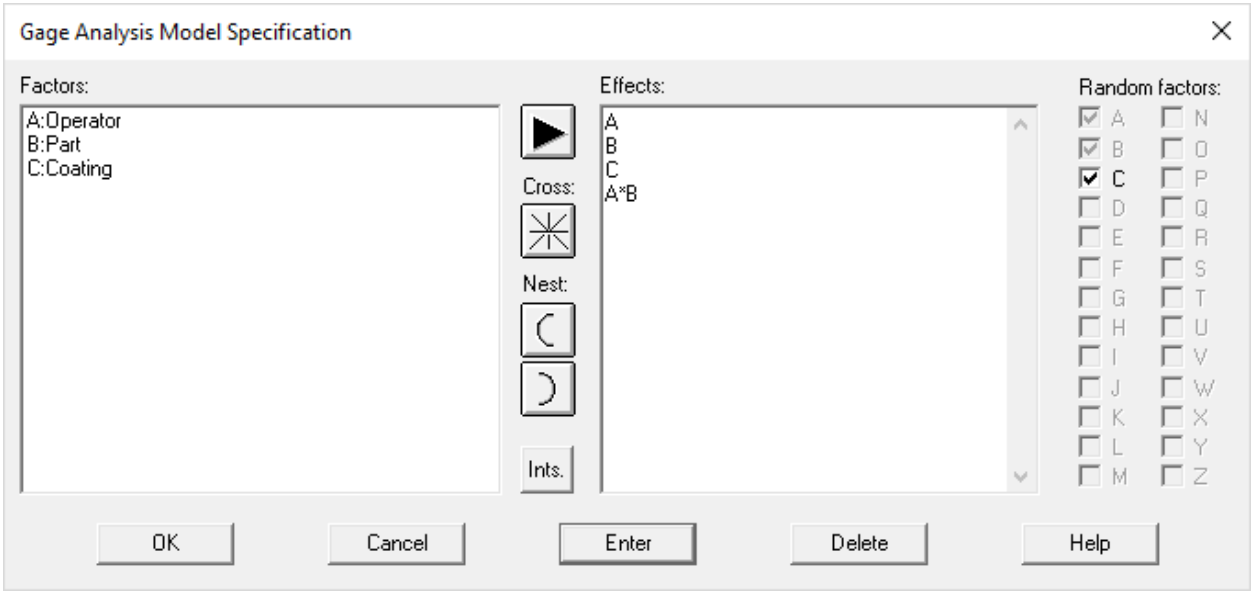
The initial dialog box displayed when the procedure is selected from the main Statgraphics menu is used to specify the names of the columns containing results from the study:



- **Operators**: numeric or non-numeric column indicating the appraiser corresponding to the measurements in each row.
- **Parts**: numeric or non-numeric column indicating the item corresponding to the measurements in each row.
- **Measurements**: numeric column containing the measurements.
- **Additional Factors**: additional factors (if any) that were varied during the study.
- **Study Header**: optional header to be printed at the top of each output table.
- **Select**: subset selection.

If the study has *m* appraisers, *n* items, and *r* trials, a complete study would consist of *mnr* rows. However, this procedure allows some of the data to be missing.

After the data columns are specified, a second dialog box is displayed on which to indicate the model to be fit to the data:



- **Factors**: Each of the factors is assigned a letter between A and Z.

- **Effects**: The effects to be included in the model are specified using the letters assigned to the factors. Effects are entered as follows:

    1. *Main effects for crossed factors* - Enter a single letter such as A.

    2. *Interactions between crossed factors* - Enter a term such as A*C to include the interaction between factors A and C or A*B*C to specify a 3-factor interaction.

    3. *Effects of nested factors* - Enter a term such as B(A) if factor B is nested within factor A or C(B A) if factor C is nested within combinations of factors A and B.

- **Random Factors:** Factors may be either *Fixed* or *Random*. A factor is *Random* if its levels consist of a random sample of levels from a population of possible levels. A factor is *Fixed* if its levels are selected by a nonrandom process or if its levels consist of the entire population of possible levels.

The effects specified on the dialog box above are:

A: the main effects of *Operator*.

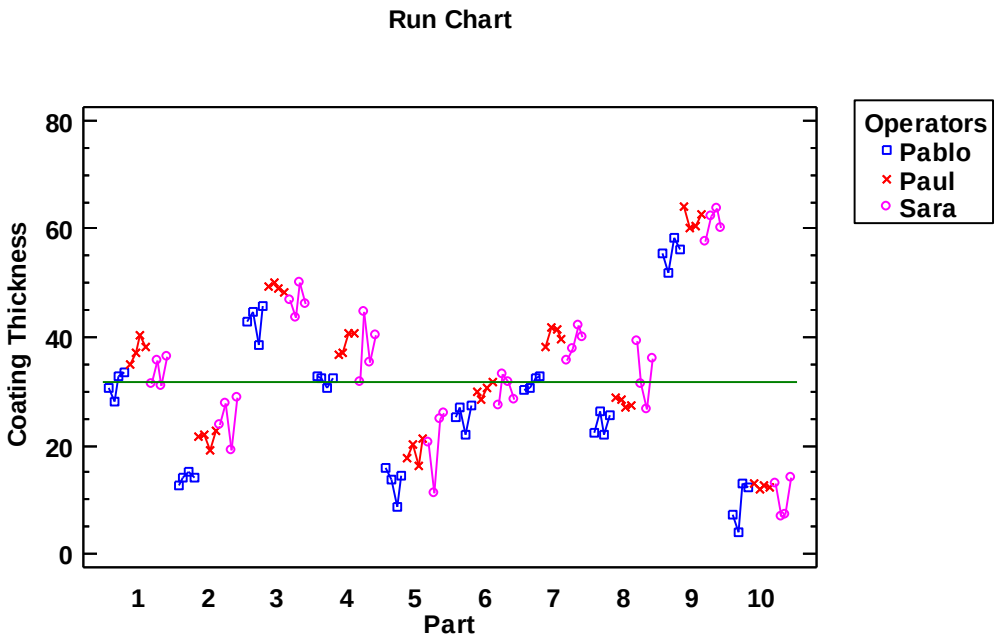
B: the main effects of *Part*.

C: the main effects of *Coating*.

*A*B*: the interactions between *Operator* and *Part*. This term will allow the *Part* effect to be different for the 3 levels of *Operator*.

Run Chart

When analyzing data from a gage study, a useful plot to examine first is the *Run Chart*.



The *Run Chart* plots each of the measurements in the study, grouped by appraiser and part. If the measurement system is capable of distinguishing one part from another, the measurements should not be randomly scattered, but should show obvious grouping by parts. This is true in the sample data above, although you can also see some differences amongst the operators.

Operator and Part Plot

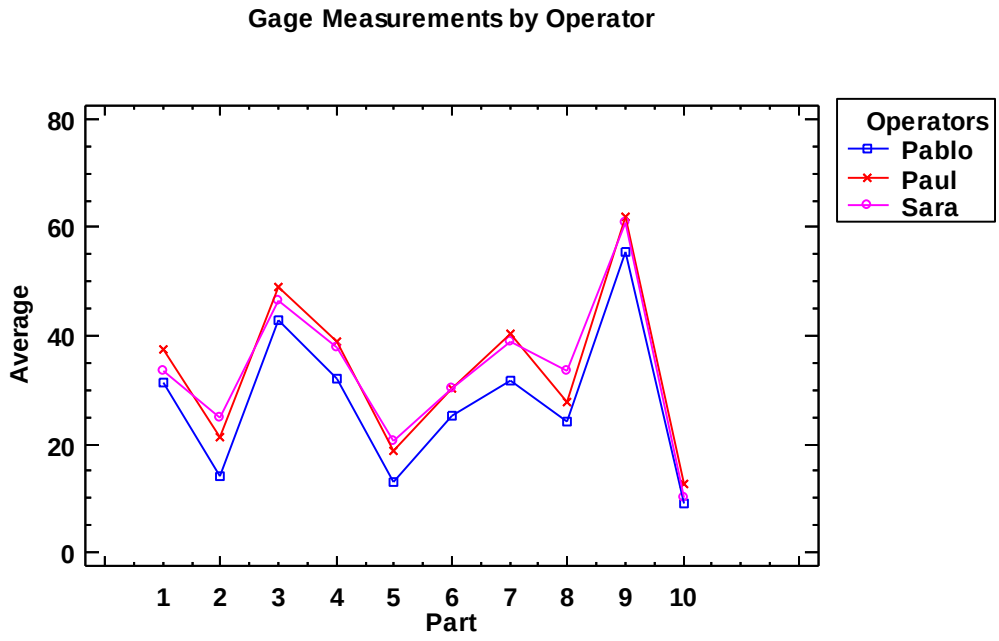
In estimating repeatability and reproducibility, the first step is to calculate the sample means for each combination of operator and part:

$\bar{x}_{ij}$ = average measurement made by operator *i* on part *j*

$\bar{x}_{i}$ = average measurement made by operator *i*

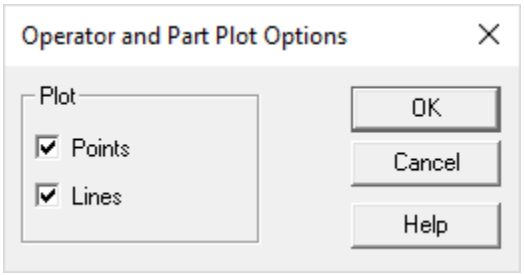
$\bar{x}$ = average of all the measurements

The *Operator and Part* plot displays $\bar{x}_{ij}$, the operator by part averages:



This plot is useful for showing any consistent differences between the appraisers. For example, Pablo appears to be consistently lower than the other 2 appraisers.

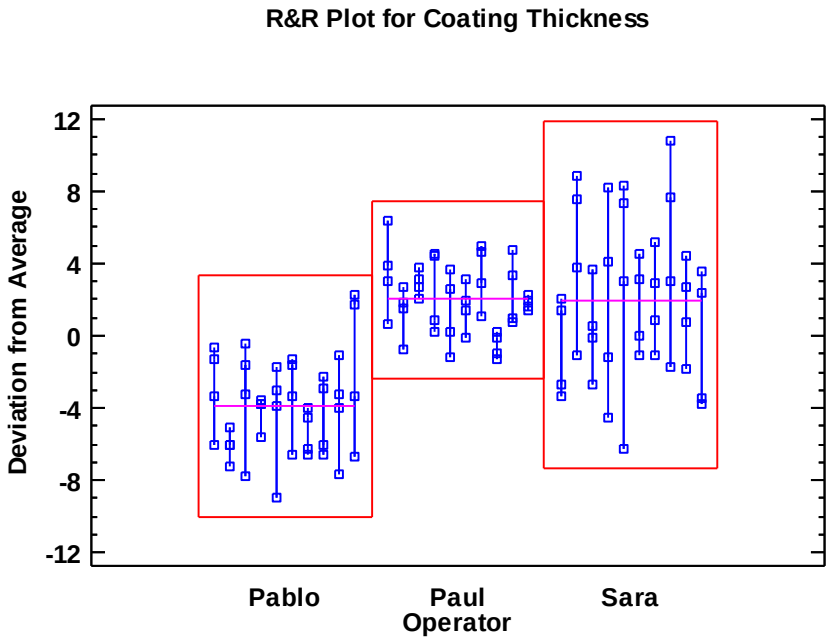
Pane Options



- **Points**: plot point symbols.
- **Lines**: connect the points with a line.

R&R Plot

Another useful plot is the *R&R Plot*, which contains a single point for each measurement in the study:



The vertical axis is scaled to show the difference between each measurement and the overall mean of all the measurements. The points are grouped by appraiser, and a horizontal line is drawn at the average deviation $\bar{x}_i - \bar{x}$ for each appraiser. Vertical lines connect measurements made by the same appraiser on the same item.

It can be seen from the above plot that Pablo's measurements are smaller on average than those of the other two appraisers. Also, Paul shows considerably less variability than the other appraisers. In fact, the repeatability of Sara is poor, since she has several large discrepancies between repeated measurements on the same item.

Analysis Summary

Having examined the data, it is then time to view the numerical results. The *Analysis Summary* displays estimates of the variability due to repeatability, reproducibility, and parts.

Gage R&R - GLM Method - Coating Thickness
Operators: Operator
Parts: Part
Measurements: Coating Thickness
Additional random factors:
Coating
3 operators 10 parts 4 trials

Gage Repeatability and Reproducibility Report

Measurement Unit	*Estimated Variance*	*Percent Contribution*	*Percent of R&R*
Total R&R	24.5857	10.80%	
Repeatability	4.87594	2.14%	19.83%
Reproducibility	19.7097	8.66%	80.17%
Operators	11.2531	4.9446%	45.7710%
Coating	6.26859	2.7544%	25.4969%
Operators*Parts	2.18804	0.9614%	8.8997%
Parts	202.998	89.1971%	
Total Variation	227.583	100%	

Number of distinct categories (ndc): 4

This summary displays:

- **Total R&R** – estimate of the overall variability attributable to both repeatability and reproducibility.
- **Repeatability** - estimate of the variation between measurements made by the same appraiser on the same part, usually attributed to the instrument.
- **Reproducibility (Operators)** - estimate of the variation between measurements made by different appraisers on the same part, usually attributed to the appraiser.
- **Reproducibility (Coating)** - estimate of the variation between measurements made on different coatings applied to the same part.
- **Reproducibility (Operators*Parts)** - estimate of the variation between measurements caused by interactions between *Operators* and *Parts*.
- **Parts** – estimate of the actual variability among the items measured. If the measurement process is capable of separating good items from bad items, this should be large compared to the variability of the measurement process.

- **Total Variation** – sum of the variability due to the measurement process and the actual variability amongst the items.

For each measurement unit (component), the columns of the table show:

- **Estimated Variance** – the estimated variance of each component $\hat{\sigma}^2_{component}$.

- **Percent Contribution** – the percentage of the total variance:

$$100\frac{\hat{\sigma}^2_{component}}{\hat{\sigma}^2_{total}}\% \quad (1)$$

where

$$\hat{\sigma}^2_{total} = \hat{\sigma}^2_{repeat} + \hat{\sigma}^2_{repro} + \hat{\sigma}^2_{parts} \quad (2)$$

- **Percent of R&R** – the percentage of the overall measurement variance:

$$100\frac{\sigma^2_{component}}{\hat{\sigma}^2_{R\&R}}\% \quad (3)$$

where

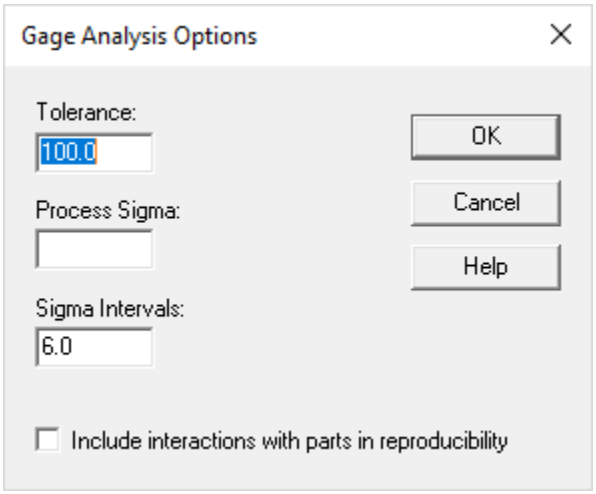
$$\hat{\sigma}^2_{R\&R} = \hat{\sigma}^2_{repeat} + \hat{\sigma}^2_{repro} \quad (4)$$

Of particular interest is $\hat{\sigma}_{R\&R}$. The rule of thumb cited by the AIAG is that if *this quantity* is less than 10% of the total variation, then the measurement system is usually deemed to be acceptable. In certain cases, values between 10% and 30% may also be acceptable, depending on the circumstances. The relative percentages of repeatability and reproducibility can also be helpful in isolating the largest source of variability in the measurement process.

One more statistic is also displayed:

- **Number of distinct categories (ndc)** – According to the AIAG (2002), *ndc* represents "the number of distinct categories that can be reliably distinguished by the measurement system." It is basically a measure of how many 97% confidence intervals for the true value being measured can fit within the range of expected part-to-part variation. Values greater than or equal to 5 are desirable.

Analysis Options



- **Tolerance**: the distance between the specification limits, USL - LSL. The value entered in this field is used by the *Tolerance Analysis*, described below, to compute a *precision-to-tolerance* (P/T) ratio.

- **Process Sigma**: the value of the process standard deviation, if known. If a value is supplied, the estimated total variation $\hat{\sigma}_{total}$ will be replaced by this value and the part-to-part variation updated accordingly. This impacts the estimated percentage contributions of all components.

- **Sigma Intervals**: the sigma multiple *K* used to compare the spread of the measurement error relative to the distance between the specifications. The value 6.0 gives 99.73% coverage based upon a normal distribution. For 99% coverage, change the value to *K = 5.15*. The default value of *K* is determined by the settings on the *Gage Studies* tab of the *Preferences* selection on the *Edit* menu.

- **Include interactions with parts in reproducibility**: whether terms representing the interaction of parts and other factors should be included in the estimate of reproducibility. If not checked, the interaction will be displayed as a separate line item.

ANOVA Table

The *GLM Method* estimates the variation components using an analysis of variance. The *ANOVA Table* shows the numerical values used in that calculation.

ANOVA Table

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Operators	927.504	2	463.752	73.0518	0.0000
Parts	22046.4	9	2449.6	385.869	0.0000
Coating	380.992	1	380.992	60.0151	0.0000
Repeatability	679.264	107	6.34826		
Total (Corr.)	24034.2	119			

The table contains a row for each component in the statistical model: variability due to operators, variability due to parts, variability due to the coating, variability due to interactions between operators and parts (if requested), and a residual term representing the repeatability of the measurement process. Of primary interest in this table are the *P-Value*s. If a P-Value is greater than or equal to 0.05, there is not a statistically significant interaction at the 5% significance level. Note that the P-Values for all terms in the above table are well below 0.05, indicating that each factor contributes significantly to the total variation.

Tolerance Analysis

An alternative method for assessing the acceptability of a measurement system is to compare the estimated measurement variation to the distance covered by the specification limits for the variable being measured. If LSL and USL represent the lower specification limit and upper specification limit, respectively, then the tolerance is defined as

$$\text{tolerance} = \text{USL} - \text{LSL} \qquad (5)$$

The *Tolerance Analysis* pane compares the estimates of the various measurement unit variations to the tolerance.

Tolerance Analysis
3 operators 10 parts 4 trials

Tolerance = 100.0

Measurement Unit	Standard Deviation	6.0 Std. Dev.	Percent of Tolerance
Total R&R	4.90178	29.4107	29.41%
Repeatability	2.51958	15.12	15.12%
Reproducibility	4.20466	25.23	25.23%
Operators	3.38158	20.2895	20.2895%
Coating	2.49881	14.9929	14.9929%
Parts	203.604	85.6140	85.6140%
Total Variation	15.0875	90.5248	90.5248%

The table shows:

- **Standard Deviation** – the square root of the estimated variance component.

- **6.0 Std. Dev.** - displays $K\hat{\sigma}$ for each of the various error components. If K equals 6.0, this estimates the interval within which the associated error component will lie 99.73% of the time.

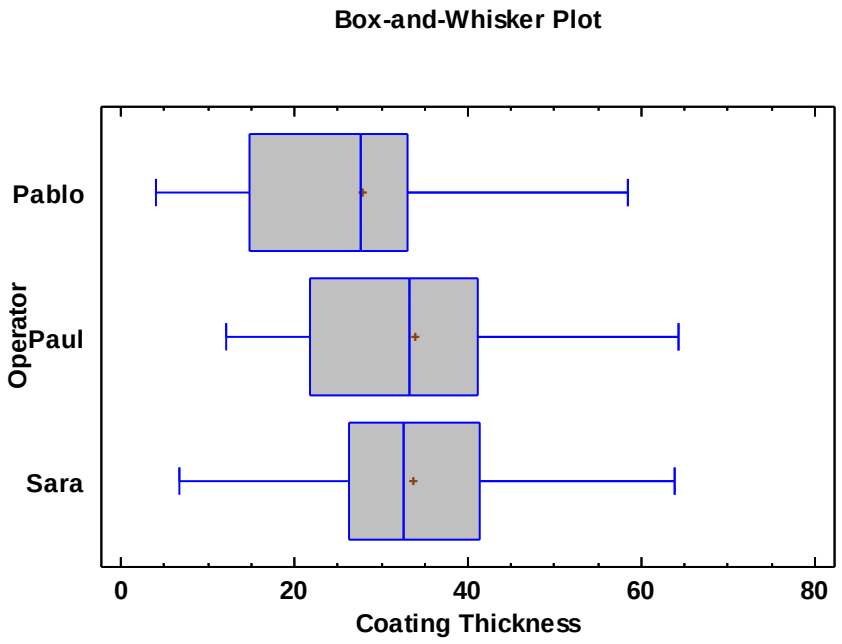
- **Percent of tolerance** – the percentage of the tolerance represented by $K\hat{\sigma}$:

$$100\frac{K\hat{\sigma}_{component}}{tolerance}\% \qquad (6)$$

Of particular interest is the *Percent of Tolerance* due to *R&R*, often called the "precision- to-tolerance ratio" or P/T. Basically, P/T is a measure of how wide the measurement error distribution is compared to the specifications for the item being measured. Values of P/T less than 10% usually imply an acceptably small measurement error, although P/T may be as high as 30% in some cases and still be acceptable.

Box and Whisker Plot

The box-and-whisker plot provides an additional comparison between the appraisers.



For each appraiser, a box-and-whisker plot is drawn as follows:

- The rectangular *box* covers the central 50% of an appraiser's measurements, ranging between the lower quartile and the upper quartile.

- A vertical line is drawn within the box at the *median* for that appraiser.

- A plus sign is drawn to indicate the *mean* measurement $\bar{x}_i$ of each appraiser.

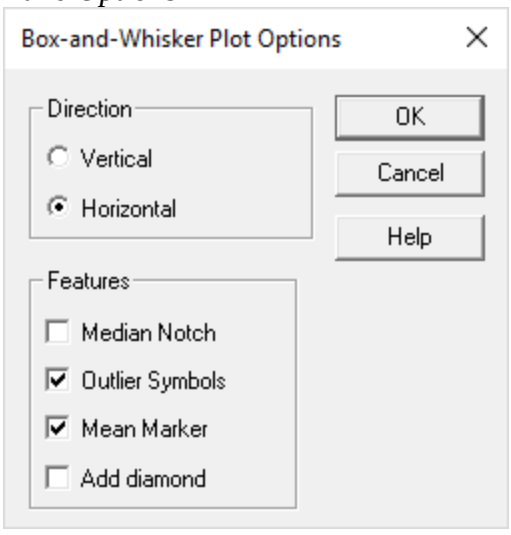
- *Whiskers* are drawn from each end of the box to the minimum and maximum value for each operator, unless outside points are detected, in which case the whiskers are drawn to the most extreme data values that are not outside points.

- Any *outside points* are indicated using point symbols such as a small square, or a square with a plus sign through it if the points are *far outside*.

For more details on outside points and other features of box-and-whisker plots, refer to the documentation for the standalone *Box-and-Whisker Plot* procedure.

In the above plot, no single measurements appear to be outliers since there are not any outside points.

Pane Options



- **Direction**: the orientation of the plot, corresponding to the direction of the whiskers.

- **Median Notch**: if selected, a notch will be added to the plot showing an approximate 100(1-α)% confidence interval for the median at the default system confidence level (set on the *General* tab of the *Preferences* dialog box on the *Edit* menu).

- **Outlier Symbols**: if selected, indicates the location of outside points.

- **Mean Marker**: if selected, shows the location of the sample mean as well as the median.

- **Add diamond:** if selected, a diamond will be added to the plot showing a 100(1-α)% confidence interval for the mean at the default system confidence level.

Calculations

The calculations are performed by the GLM (General Linear Models) procedure.