Capability Analysis Using Statgraphics Centurion

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Outline

- Definition of process capability analysis
- Examples
  1. Capability analysis for attributes
  2. Estimating capability for variable data
  3. Capability indices
  4. Statistical tolerance limits
  5. Multivariate capability analysis
- Sample size determination

Capability Analysis

Determination based on data of a process's ability to meet established specifications.

Specifications may be stated in terms of variables (such as the tolerance on the diameter of a part) or in terms of attributes (such as the frequency of customer complaints).

Capability Measurements

The essential measure of process capability is DPM (defects per million) or DPMO (defects per million opportunities), defined as the number of times that a process does not meet the specifications out of every million possibilities.

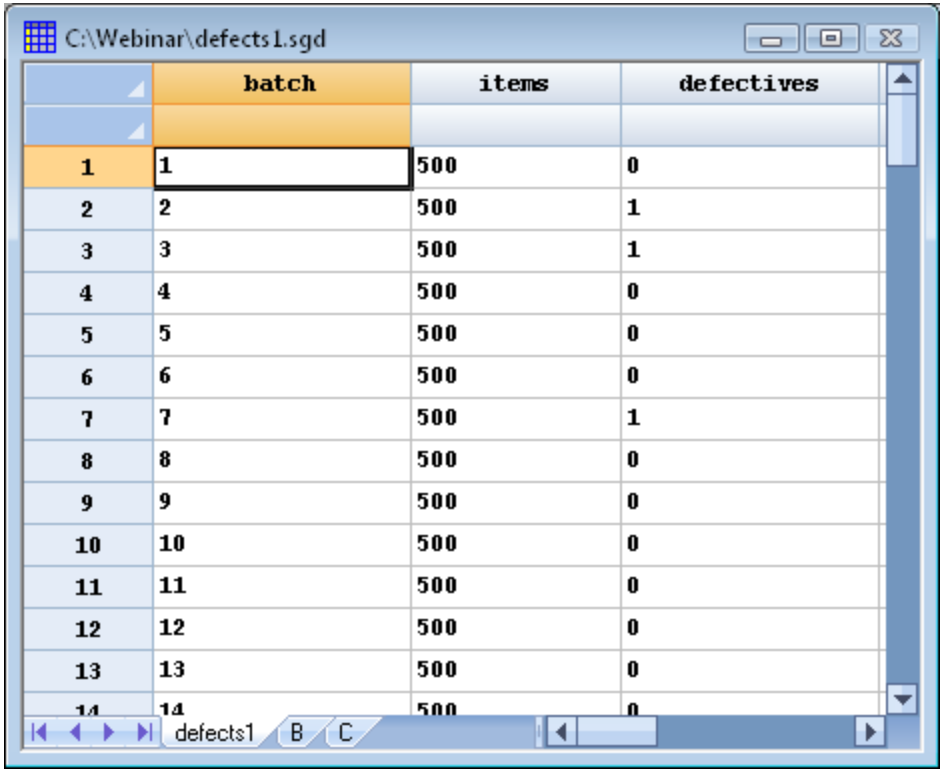
DPM may be estimated directly or inferred from statistics such as a capability index or statistical tolerance limit.

How can we estimate capability using Statgraphics?

- Direct counting of defects
- Estimation of DPM from a fitted distribution
- Indirect inference about DPM from a capability index
- Demonstration of required capability through a statistical tolerance interval or bound

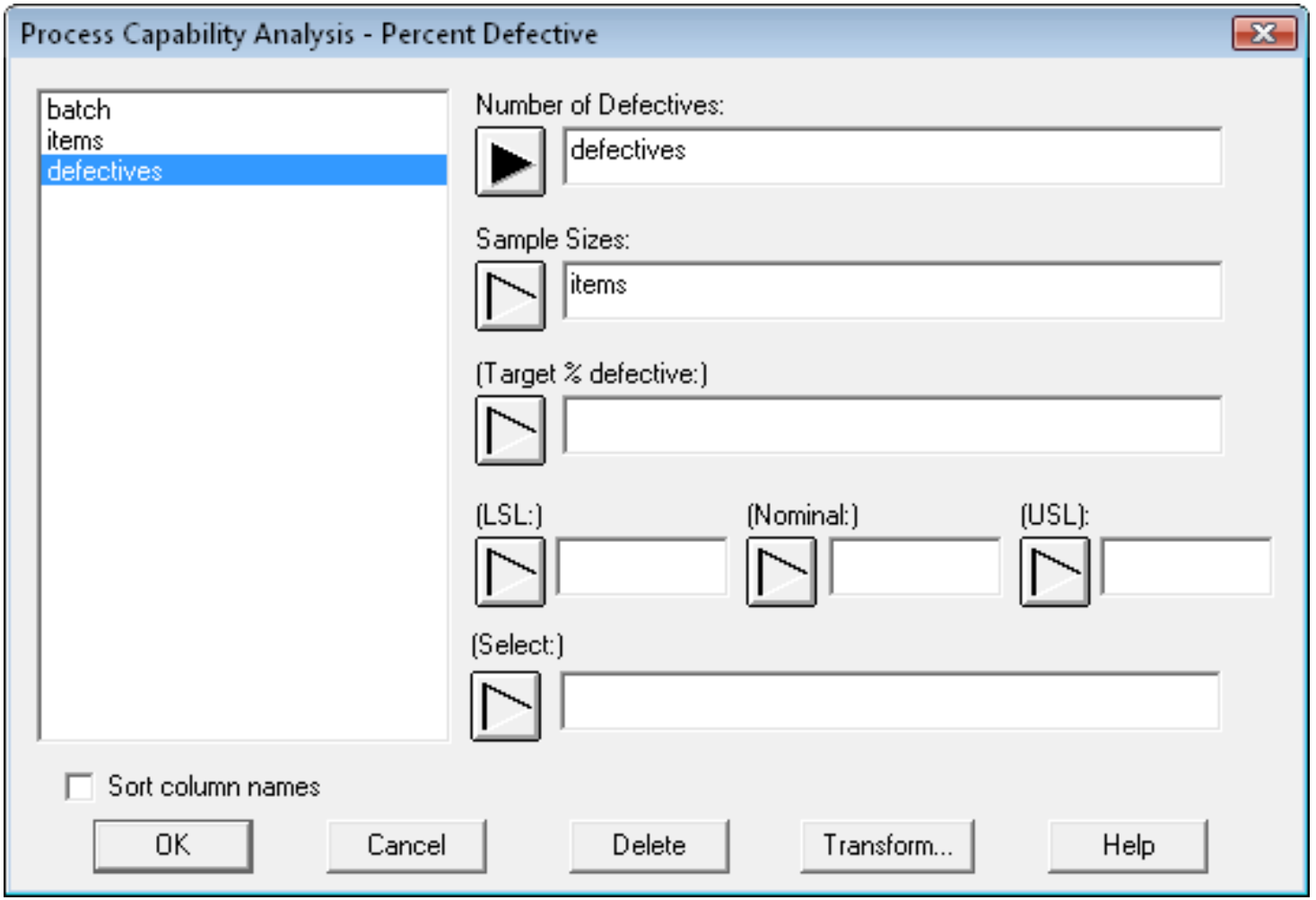
Example #1 - defects1.sgd

Inspected k=30 batches of n=500 items each. Counted number of defective items.



	batch	items	defectives
1	1	500	0
2	2	500	1
3	3	500	1
4	4	500	0
5	5	500	0
6	6	500	0
7	7	500	1
8	8	500	0
9	9	500	0
10	10	500	0
11	11	500	0
12	12	500	0
13	13	500	0
14	14	500	0

Procedure Capability Analysis – Percent Defective



Output

Process Capability Analysis (Percent Defective) - defectives

Process Capability Analysis (Percent Defective) - defectives

Data variable: defectives

Distribution: Binomial
number of samples = 30
average sample size = 500.0
mean percent defective = 0.0866667

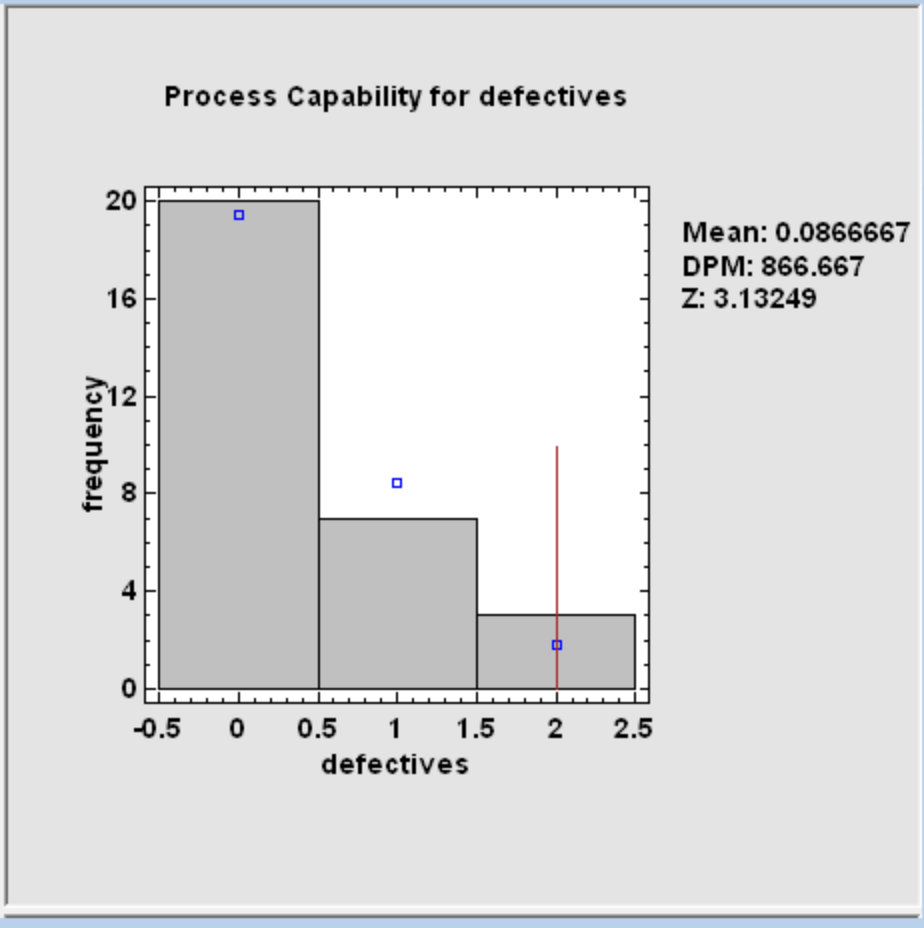
	Estimate	*Upper 95% Bound*
Mean percent defective	0.0866667	0.13772
Defects per million	866.667	1377.2
Process Z	3.13249	2.9939
Tolerance limits (average size sample)		2

The StatAdvisor

The 30 values of defectives have an average of 0.0866667% defective items. This equ
confidence bound indicates that the mean percent of defective items in the population

The process Z value converts the estimated mean percent defective to a capability ind
capability of variable data. In most cases, a Z value of at least 4 is desirable.

The tolerance limits show the likely variability amongst samples in the population. In
expected to have no more than 2.0 defective items.



Conclusions

- Best estimate for DPM = 866.7
- With 95% confidence, DPM is no greater than 1,377.2
- Tolerance limit: 95% of all batches of n=500 items will have no more than 2 defectives
- Equivalent Z = 3.13

Example #2 - resistivity.sgd

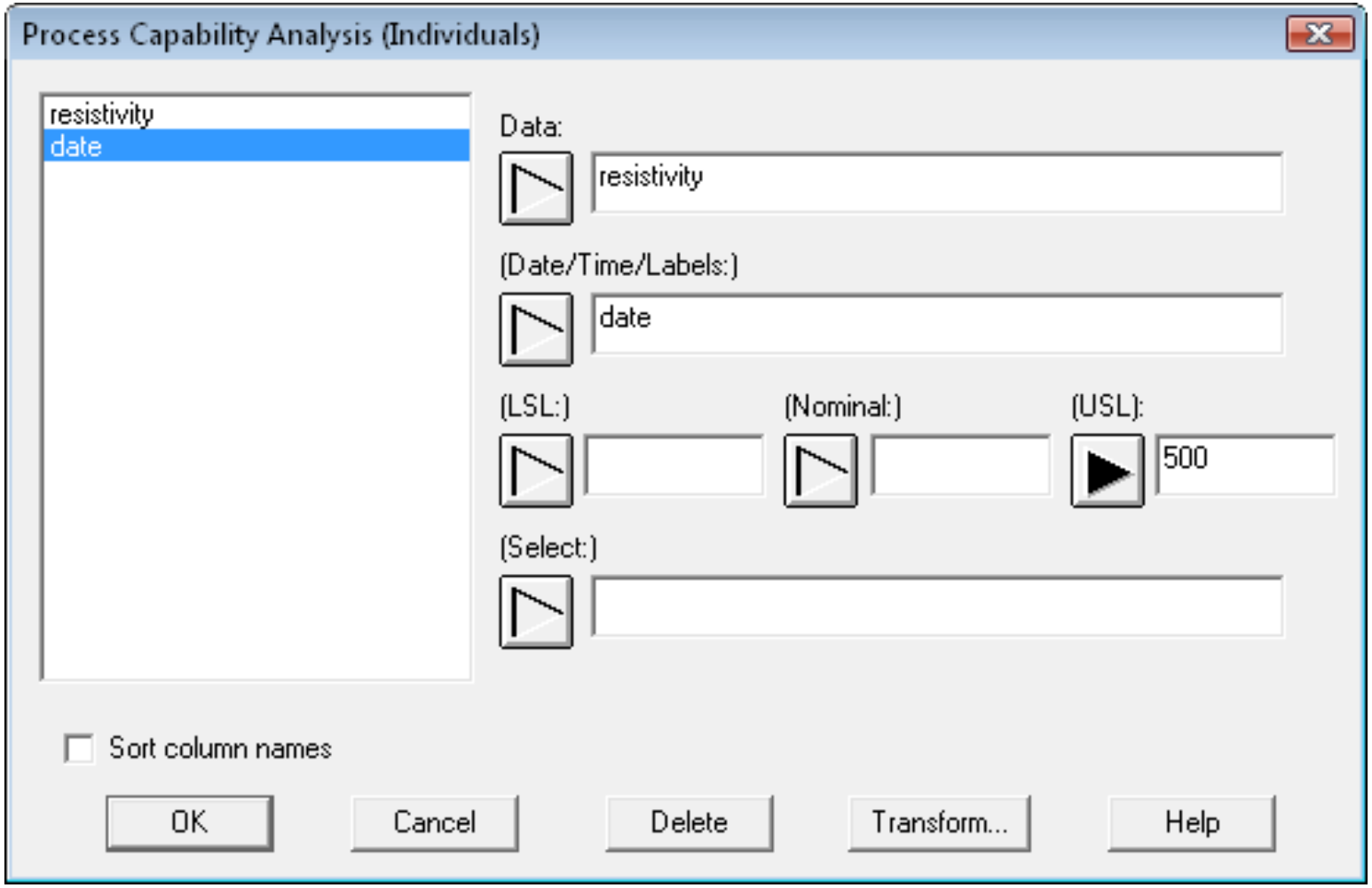
Measured resistivity of n=100 electronic components

resistivity.sgd

	resistivity	date
1	211.2	1/1/07
2	435.0	1/2/07
3	154.9	1/3/07
4	186.8	1/4/07
5	292.3	1/5/07
6	155.4	1/6/07
7	255.7	1/7/07
8	165.8	1/8/07
9	227.9	1/9/07
10	178.8	1/10/07
11	279.8	1/11/07
12	267.5	1/12/07
13	297.4	1/13/07
14	233.7	1/14/07
15	156.7	1/15/07
16	181.1	1/16/07

resistivity B C

Procedure Capability Analysis – Variable Data - Individuals



Selecting Proper Distribution

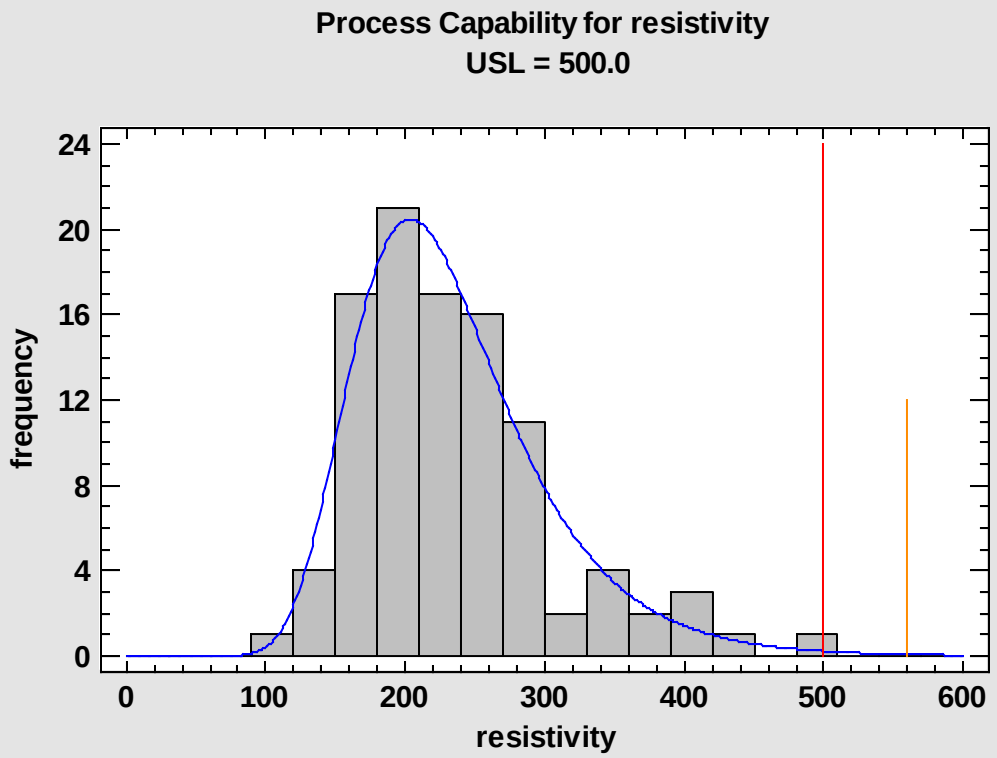
Tests for Normality for resistivity

Test	*Statistic*	*P-Value*
Shapiro-Wilk W	0.924904	0.00000550813

Comparison of Alternative Distributions

Distribution	*Est. Parameters*	*Log Likelihood*	*KS D*
Largest Extreme Value	2	-557.918	0.0422297
Lognormal	2	-558.632	0.0444956
Loglogistic	2	-559.663	0.0491967
Gamma	2	-560.767	0.0559315
Logistic	2	-566.28	0.0717518
Laplace	2	-568.183	0.0830668
Normal	2	-568.981	0.0891378
Weibull	2	-570.627	0.101169
Smallest Extreme Value	2	-593.519	0.174701
Exponential	1	-646.066	0.423526
Pareto	1	-810.833	0.585075

Capability Plot



Estimate of DPM

Transformation: none

Distribution: Largest Extreme Value
sample size = 100
mode = 203.355
scale = 53.9342
(mean = 234.486)
(sigma = 69.1733)

Equivalent 6.0 Sigma Limits
99.865 percentile = 559.698
median = 223.122
0.134996 percentile =101.514

	Observed		*Estimated*	*Defects*
Specifications	*Beyond Spec.*	*Z-Score*	*Beyond Spec.*	*Per Million*
USL = 500.0	1.000000%	2.65	0.407788%	4077.88
Total	1.000000%		0.407788%	4077.88

Capability Indices

$$C_P = \frac{USL - LSL}{6\hat{\sigma}}$$

$$C_{PK} = \min\left[\frac{\hat{\mu} - LSL}{3\hat{\sigma}}, \frac{USL - \hat{\mu}}{3\hat{\sigma}}\right]$$

$$Z = \min\left[\frac{\hat{\mu} - LSL}{\hat{\sigma}}, \frac{USL - \hat{\mu}}{\hat{\sigma}}\right]$$

Long-term and Short-term

Capability Indices for resistivity

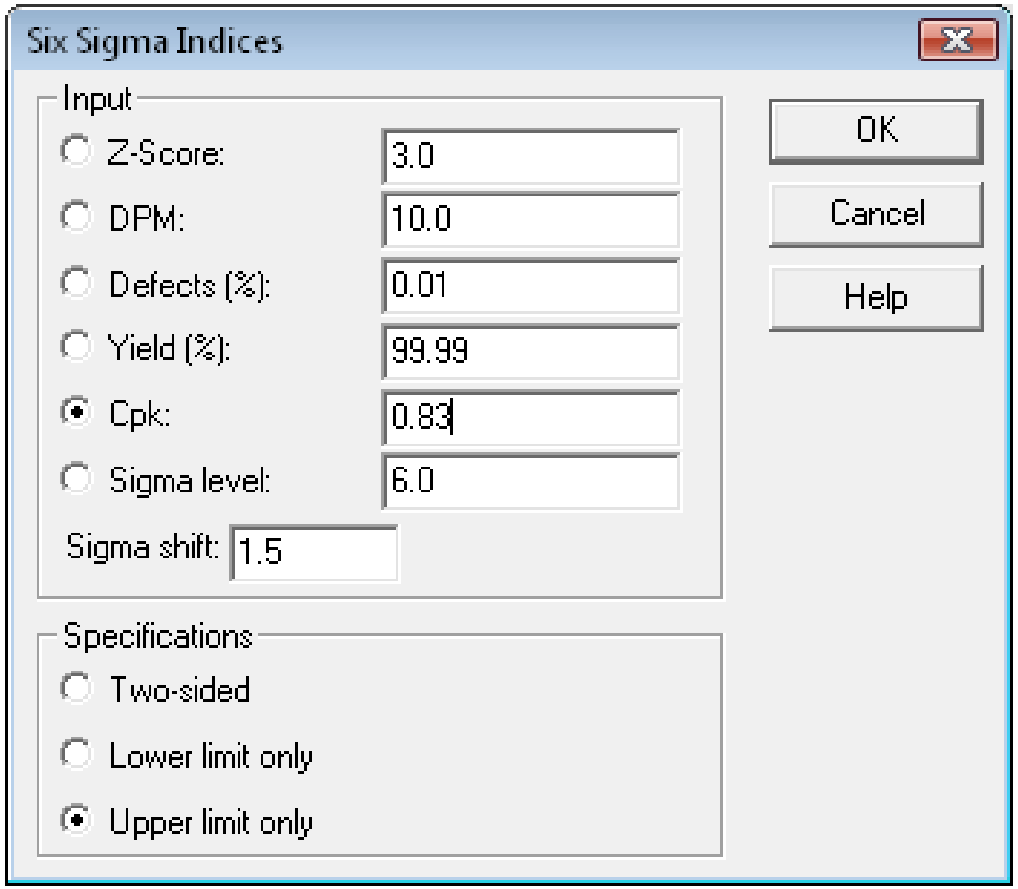
Specifications

USL = 500.0

	Short-Term	*Long-Term*
	Capability	*Performance*
Sigma (after normalization)	1.03374	1.0
Zmin	2.5592	2.64556
Cpk/Ppk	0.853067	0.881852
DPM	5245.67	4077.88

Based on 6.0 sigma limits in the normalized metric. Short-term sigma estimated from average moving range.

Six Sigma Calculator



Six Sigma Calculator

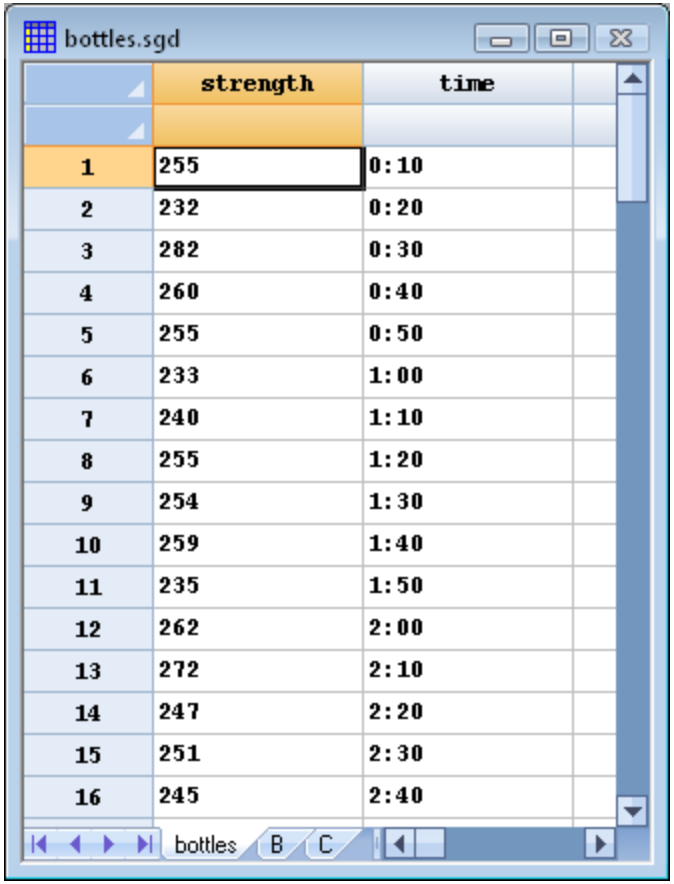
Input: Cpk = 0.83
Sigma shift = 1.5

Equivalent values:

Index	*Value*
Z-Score	2.49
DPM	6387.15
Defects	0.638715
Yield	99.3613
Cpk	0.83
SQL	3.99

Example #3 - bottles.sgd

Measured breaking strength of n=100 glass bottles

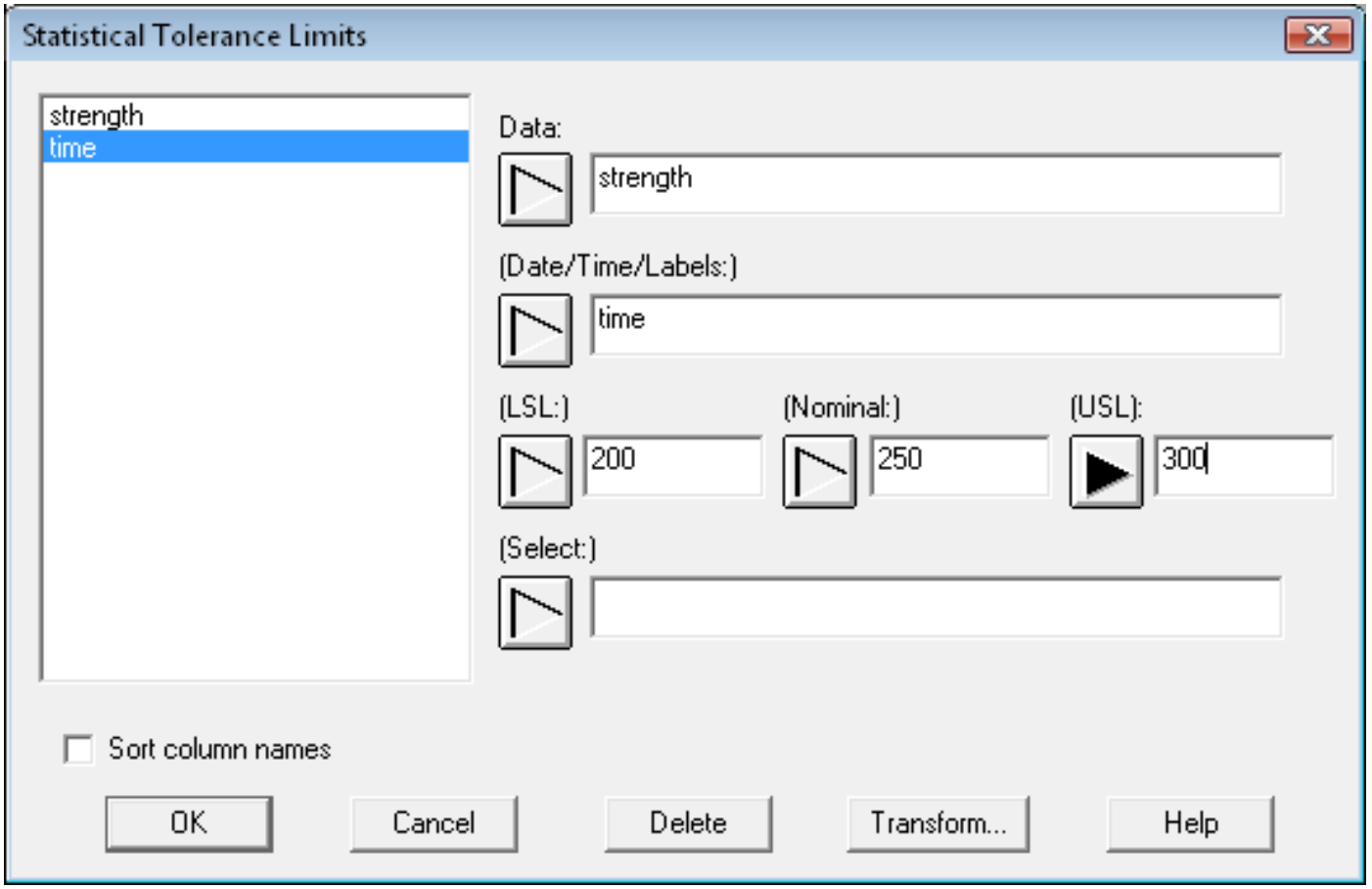


bottles.sgd

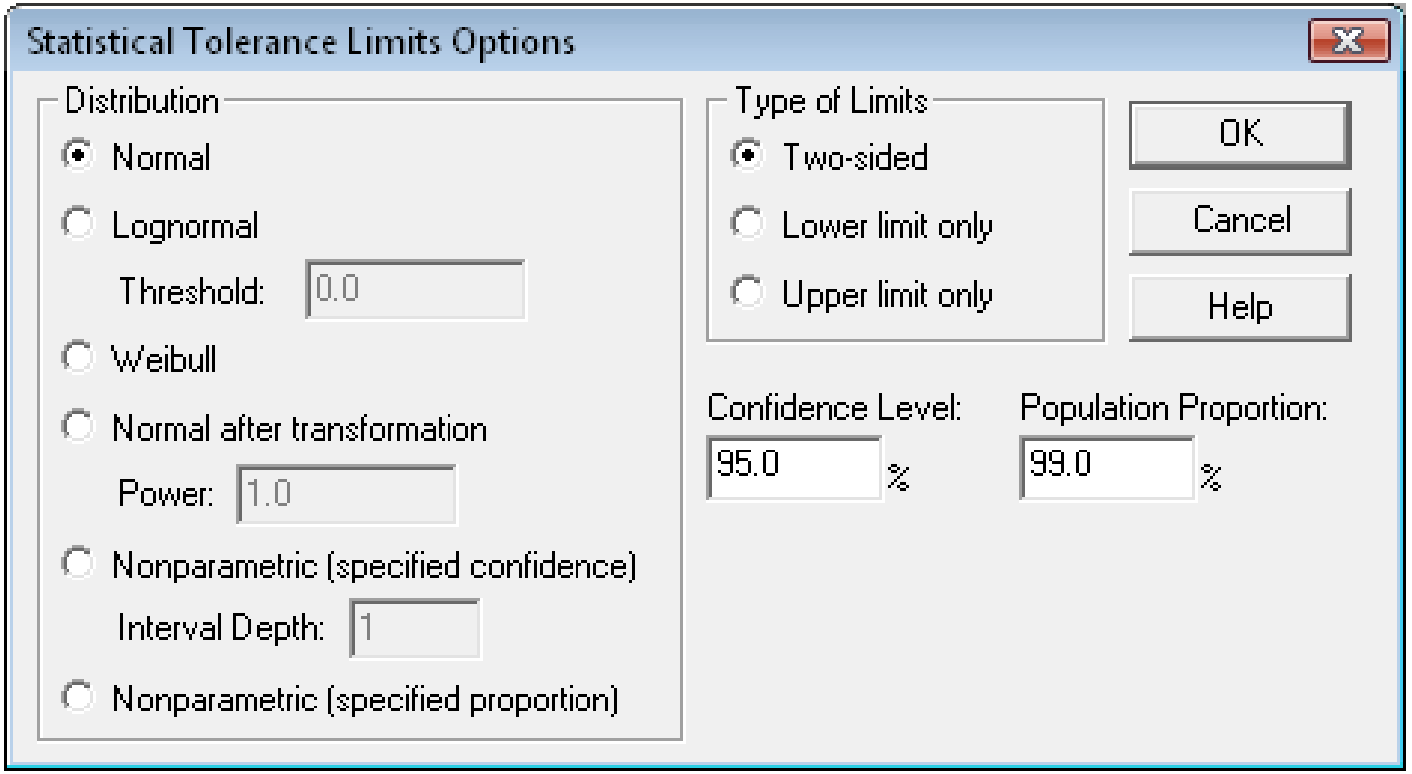
	strength	time
1	255	0:10
2	232	0:20
3	282	0:30
4	260	0:40
5	255	0:50
6	233	1:00
7	240	1:10
8	255	1:20
9	254	1:30
10	259	1:40
11	235	1:50
12	262	2:00
13	272	2:10
14	247	2:20
15	251	2:30
16	245	2:40

bottles B C

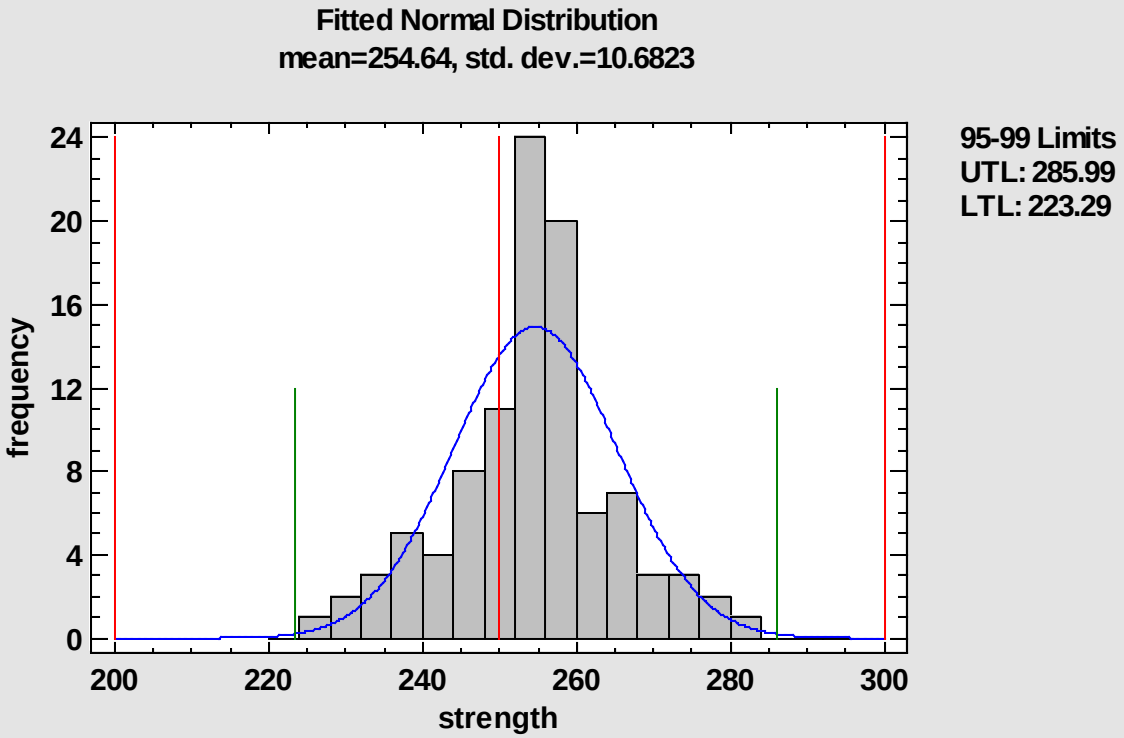
Statistical Tolerance Limits



Tolerance Limit Options



Output


Fitted Normal Distribution
mean=254.64, std. dev.=10.6823
95-99 Limits
UTL: 285.99
LTL: 223.29
frequency
0
4
8
12
16
20
24
200
220
240
260
280
300
strength

Conclusions

The StatAdvisor
Assuming that strength comes from a normal distribution, the tolerance limits state that we can be 95.0% confident that 99.0% of the distribution lies between 223.295 and 285.985. This interval is computed by taking the mean of the data +/-2.93431 times the standard deviation.

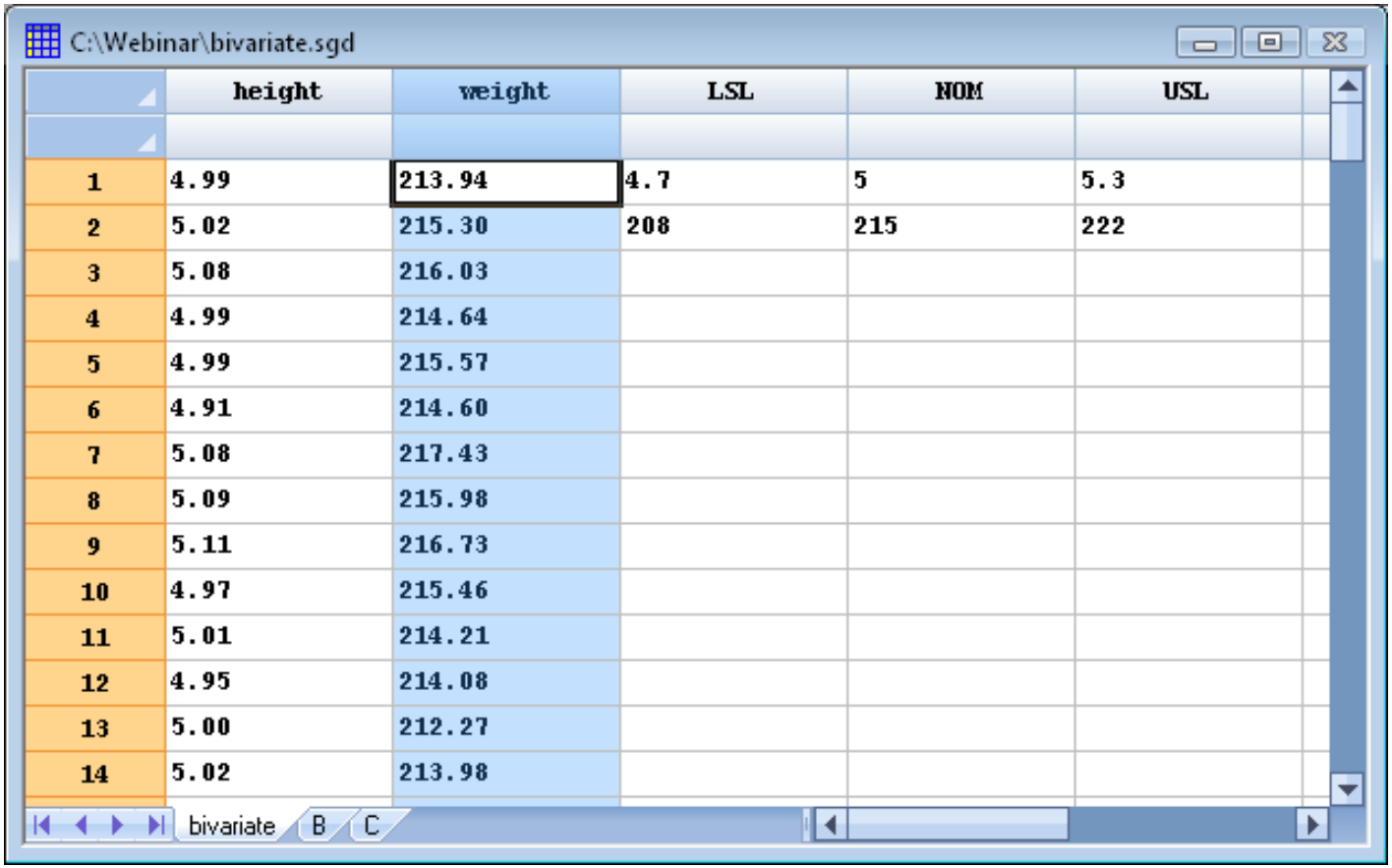
Multivariate Capability Analysis

- For multiple variables, determines the probability that ALL variables meet their established specification limits.

- Important when the variables are strongly correlated.

Example #4 - bivariate.sgd

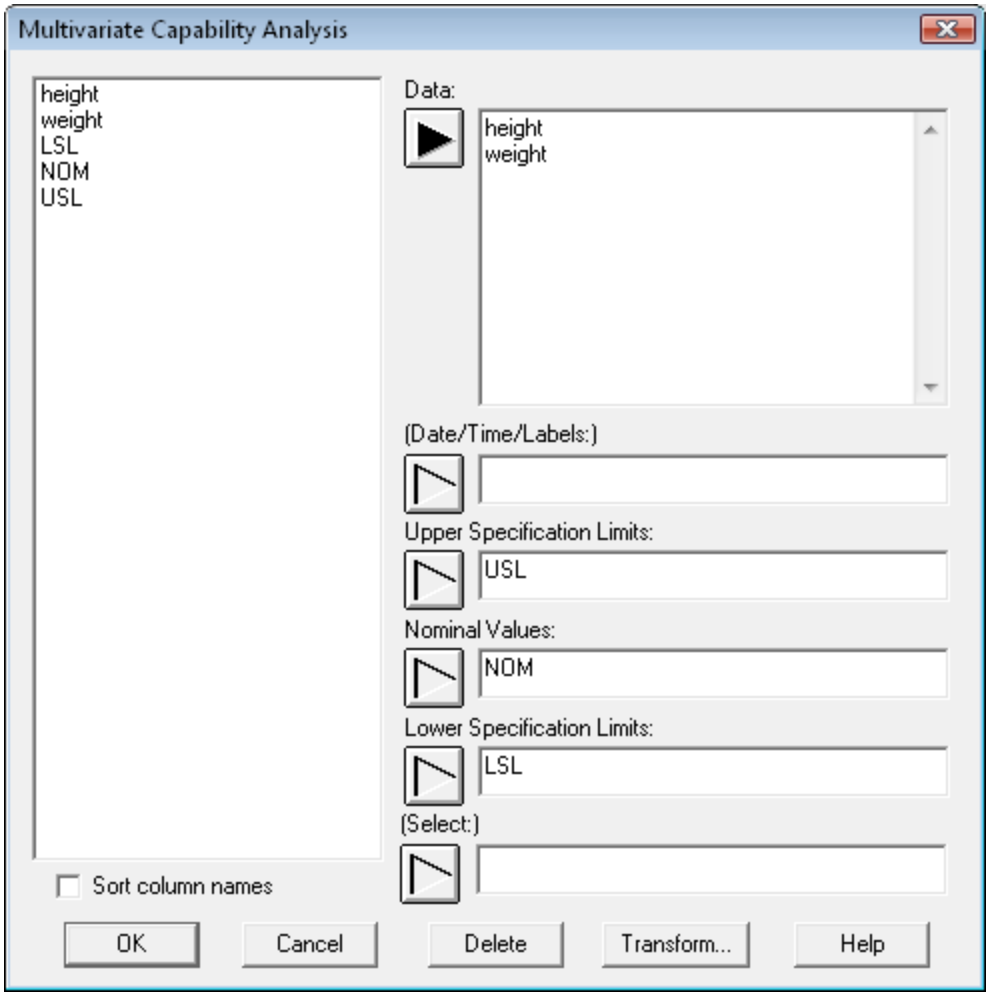
Measurements of height and weight of n=150 items.
Specs: height 5 ± 0.3, weight 215 ± 7


C:\Webinar\bivariate.sgd

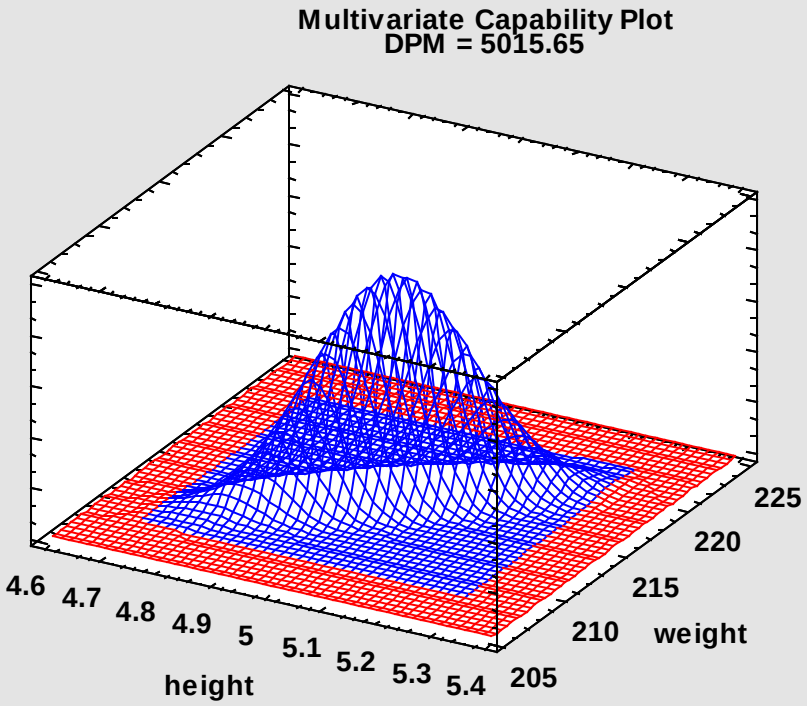
	height	weight	LSL	NOM	USL
1	4.99	213.94	4.7	5	5.3
2	5.02	215.30	208	215	222
3	5.08	216.03			
4	4.99	214.64			
5	4.99	215.57			
6	4.91	214.60			
7	5.08	217.43			
8	5.09	215.98			
9	5.11	216.73			
10	4.97	215.46			
11	5.01	214.21			
12	4.95	214.08			
13	5.00	212.27			
14	5.02	213.98			

bivariate B C

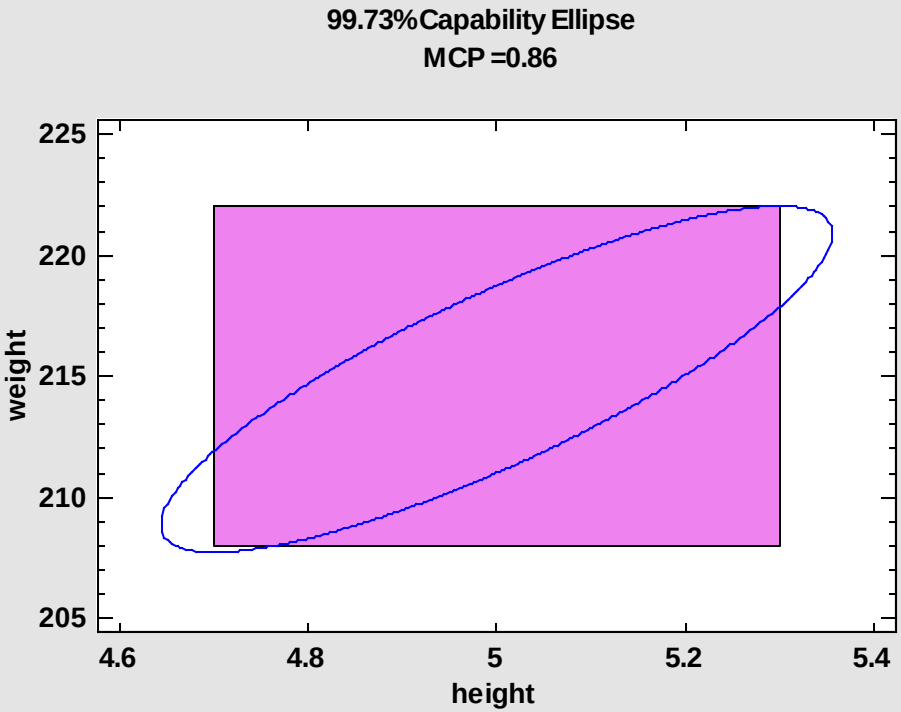
Data Input


Multivariate Capability Analysis
height
weight
LSL
NOM
USL
Data:
height
weight
(Date/Time/Labels:)
Upper Specification Limits:
USL
Nominal Values:
NOM
Lower Specification Limits:
LSL
(Select:)
Sort column names
OK
Cancel
Delete
Transform...
Help

Bivariate Normal Distribution



Capability Ellipse



Multivariate Capability

Multivariate Capability Analysis

Data variables:
- height
- weight

Number of complete cases: 150

	Sample	*Sample*			
Variable	*Mean*	*Std. Dev.*	*LSL*	*Nominal*	*USL*
height	5.0	0.105271	4.7	5.0	5.3
weight	214.883	2.11459	208.0	215.0	222.0

	Observed	*Estimated*	*Estimated*
Variable	*Beyond Spec.*	*Beyond Spec.*	*DPM*
height	0.0%	0.437478%	4374.78
weight	0.0%	0.0948817%	948.817
Joint	0.0%	0.501565%	5015.65

Capability Indices

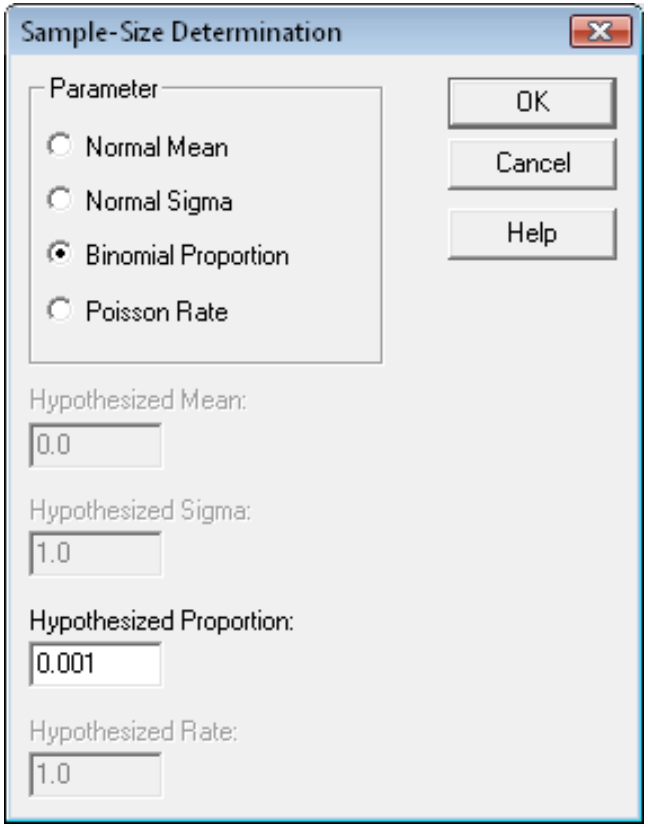
Index	*Estimate*
MCP	0.86
MCR	116.52
DPM	5015.65
Z	2.57475
SQL	4.07475

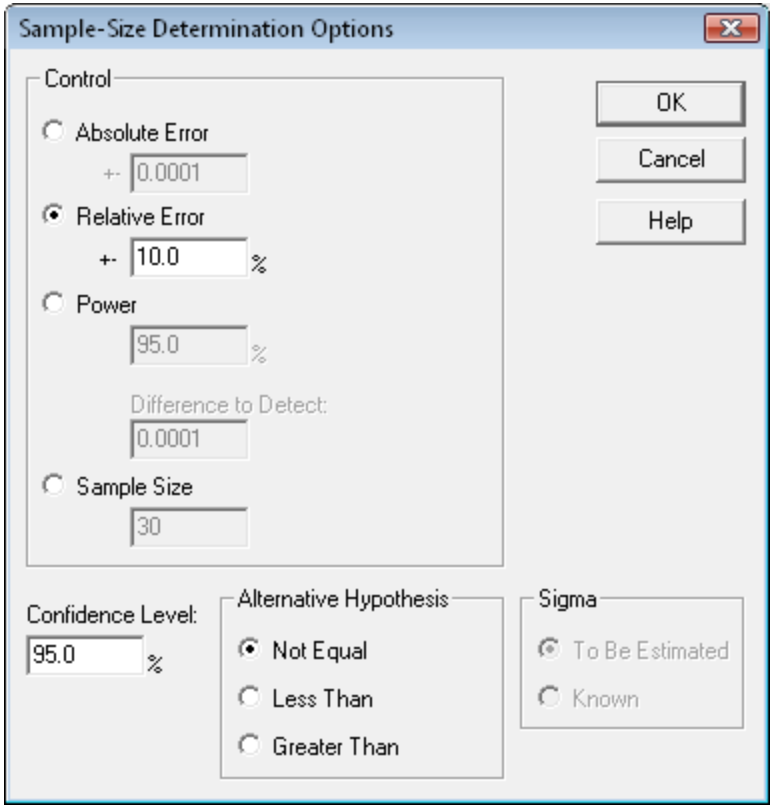
Based on 6.0 sigma limits.

Multivariate capability indices defined to give same relationship with DPM as in univariate case.

Sample Size Determination - Counting

Suppose I wish to estimate DPM to within +/-10% with 95% confidence.





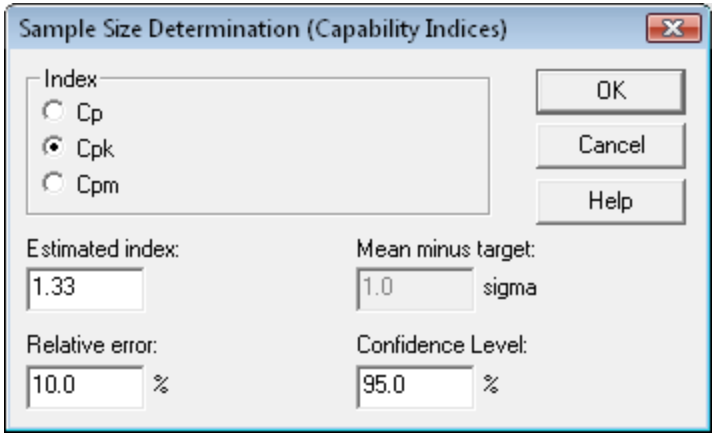
Sample-Size Determination

Parameter to be estimated: binomial parameter
Desired tolerance: +- 10.0% when proportion = 0.001
Confidence level: 95.0%

The required sample size is n=422065 observations.

Sample Size Determination – Capability Indices

Suppose I wish to estimate Cpk to within +/-10% with 95% confidence.



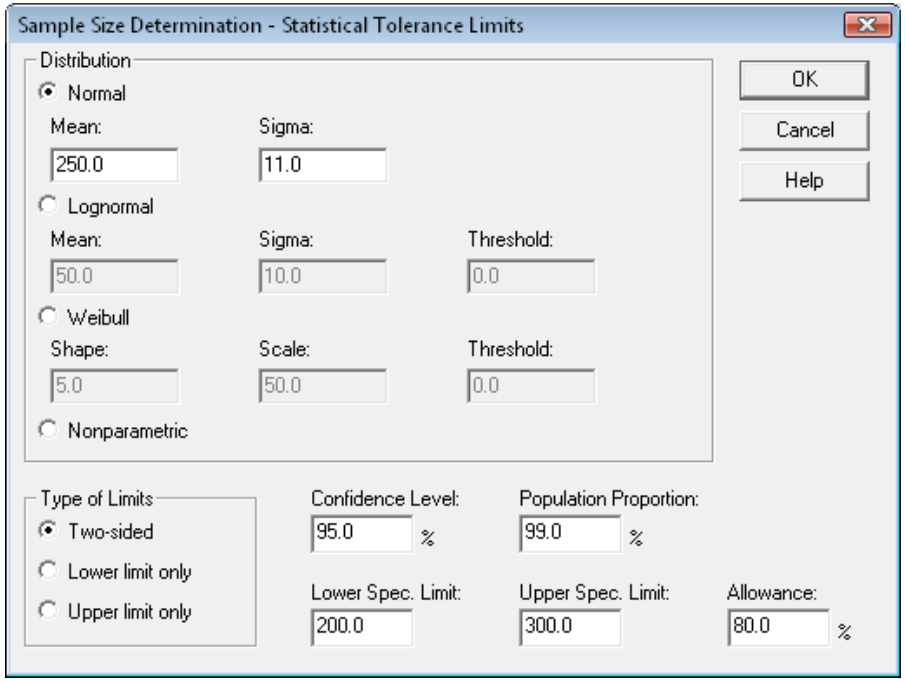
Sample Size Determination (Capability Indices)

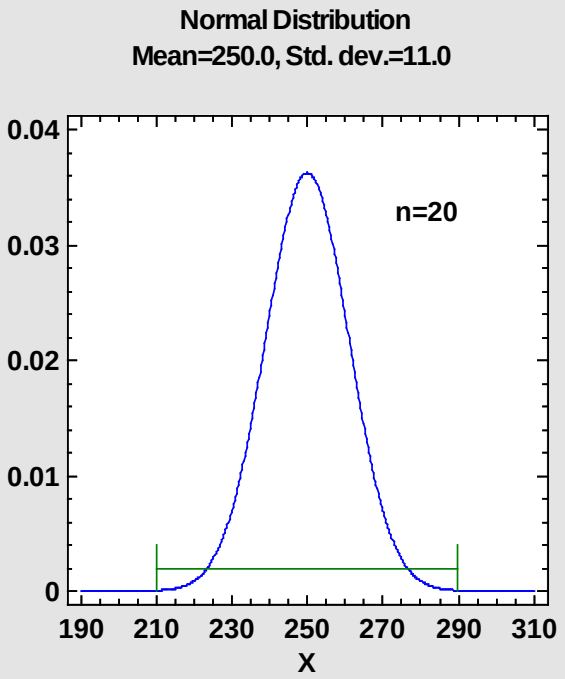
Capability index: Cpk
Estimate: 1.33
Relative error: 10.0%
Confidence level: 95.0%

The required sample size is 154.

Requires measuring n = 154 items

Sample Size Determination – Statistical Tolerance Limits





A 95-99 tolerance interval covering 80% of the distance between the spec limits requires a sample of n = 20 items in this case.

More Information

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