Capability Control Chart for Variables

statgraphics 18®
centurion

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Summary

This procedure constructs Phase II statistical process control charts for monitoring capability indices such as C_p and C_{pk}. Given a process that is deemed to be capable of satisfying stated requirements based on the analysis of variable data, these charts monitor continued compliance with those requirements.

Sample StatFolio: *capcontrolvars.sgp*

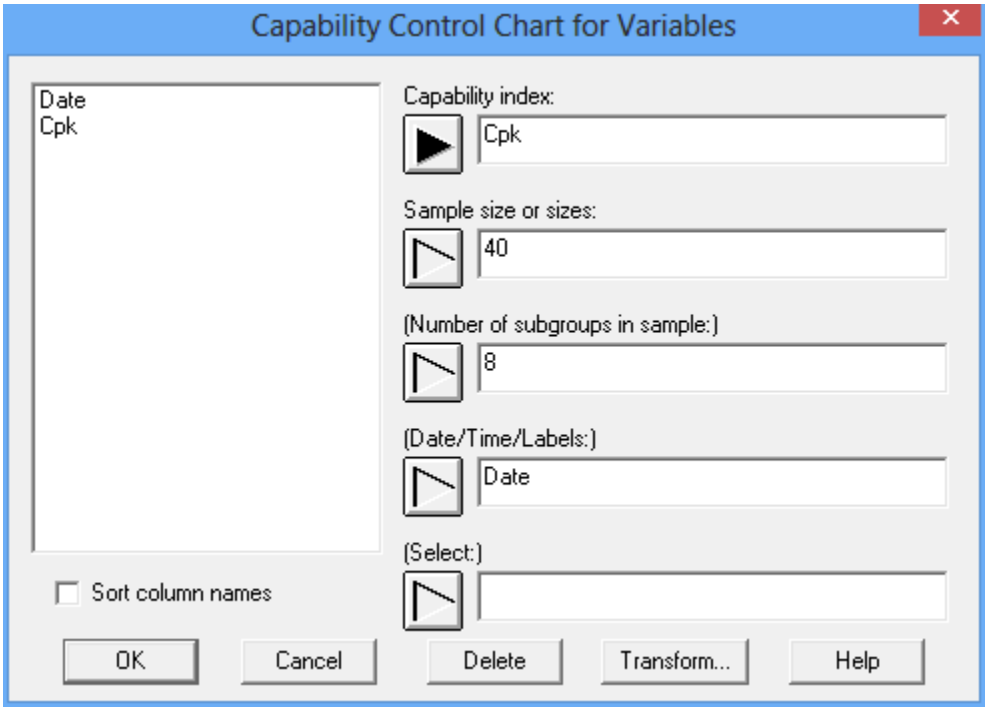
Sample Data

The datasheet contains values of C_{pk} calculated from a process believed to be in control at a C_{pk} value equal to 2.0. For each of 30 days, a sample of 5 items was collected from a manufacturing process once every 3 hours and measured. At the end of each day, the n = 40 measurements were used to calculate a C_{pk} for that day. A portion of the data is shown below:

Date	Cpk
11/1/2016	2.11
11/2/2016	1.92
11/3/2016	1.87
11/4/2016	1.97
11/5/2016	2.00
11/6/2016	1.97
11/7/2016	1.79
11/8/2016	1.83
11/9/2016	2.15
11/10/2016	1.65
…	

Data Input

The data input dialog box requests the names of the column containing the data values to be plotted and information about the size of the samples:



- **Capability index:** name of a numeric column containing the values to be plotted on the control chart. The data may consist of values for C_p, P_p, C_{pk} or P_{pk}.

- **Sample size or size:** The size of the sample corresponding to each statistic. If all data come from samples of the same size, only a single value needs to be entered. Otherwise, the name of a column with sizes for each sample should be entered.

- **Number of subgroups in sample:** the number of subgroups collected to create each sample. If all data come from samples with the same number of subgroups, only a single value needs to be entered. Otherwise, the name of a column with numbers of subgroups for each sample should be entered. Note: if data were collected as individuals, leave this field blank.

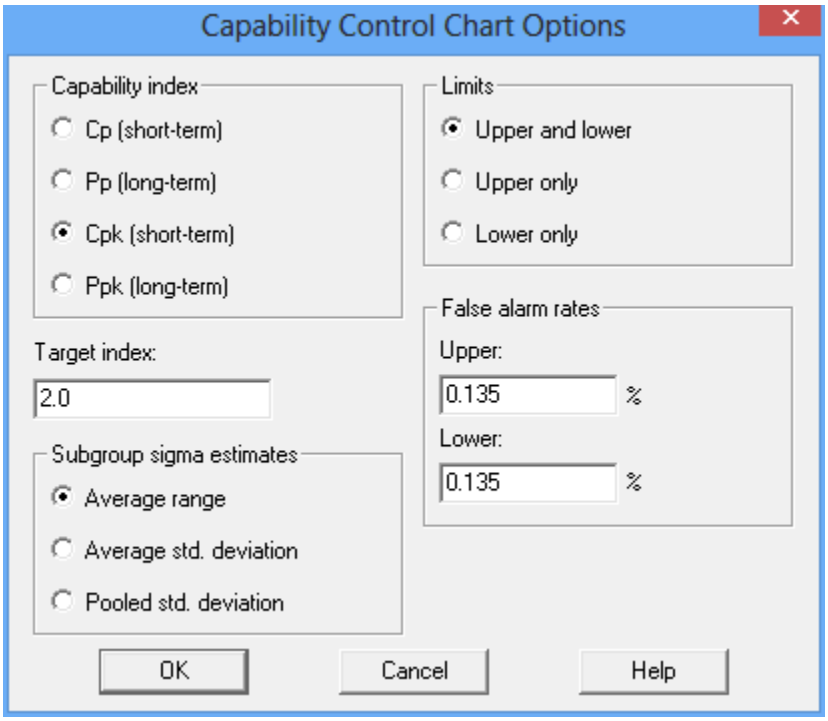
- **Date/Time/Labels:** a column with optional identifiers for each row of data. This forms the X axis of the control chart. If no entry is made, row numbers will be used to identify each sample.

- **Select:** optional subset selection.

For the example data, each day's sample has a total of 40 observations divided into 8 subgroups.

Analysis Options

The *Analysis Options* dialog box specifies the type of data entered:



- **Parameter:** specifies the type of data entered in the *Capability index* field of the data input dialog box.

- **Target index:** specifies the standard value of the index when the process is behaving as expected. This value is used as the centerline of the control chart.

- **Subgroup sigma estimate:** for the indices C_p and C_{pk} with data taken from more than 1 subgroup, specifies the method used to estimate the short-term sigma within each sample.

- **Limits:** whether the chart contains both an upper control limit and a lower control limit or only one.

- **False alarm rate:** the probability of getting a point above the upper control limit or below the lower control limit if the true index equals the *Target index*. Standard "3-sigma" control charts have a false alarm rate equal to 0.135% on each side.

Analysis Summary

The *Analysis Summary* shows the location of the control limits for the capability control chart:

Capability Control Chart for Variables - Cpk
Data variable: Cpk

Parameter: Cpk
Target: 2.0

Control Limits

Limit	*Location*	*Alpha*
UCL	3.04702	0.00135
LCL	1.47471	0.00135

Suppose the index being monitored is C_p and that the established standard for it is $C_{p,0}$. The upper control limit is located at

$$UCL = C_{P,0}\sqrt{\frac{\upsilon}{\chi^2_{1-\alpha/2,\upsilon}}} \quad (1)$$

while the lower control limit is located at

$$LCL = C_{P,0}\sqrt{\frac{\upsilon}{\chi^2_{\alpha/2,\upsilon}}} \quad (2)$$

where $\chi^2_{p,\nu}$ is the value of the chi-square distribution with ν degrees of freedom which is exceeded with probability equal to p and ν is the degrees of freedom used to estimate sigma. A similar calculation is used for P_p.

If the index being monitored is C_{pk}, the upper control limit is positioned by solving the folowing equation for UCL:

$$UCL = C_{pk,0}\left[1 + Z_{\frac{\alpha}{2}}\sqrt{\frac{1}{9m\bar{n}\,UCL^2} + \frac{1}{2\nu}}\right] \quad (3)$$

where m is the number of subgroups used to estimate each value of C_{pk}, $\bar{n}$ is the average subgroup size, and ν is the degrees of freedom used to estimate sigma. To find the lower control limit, the following equation is solved for LCL:

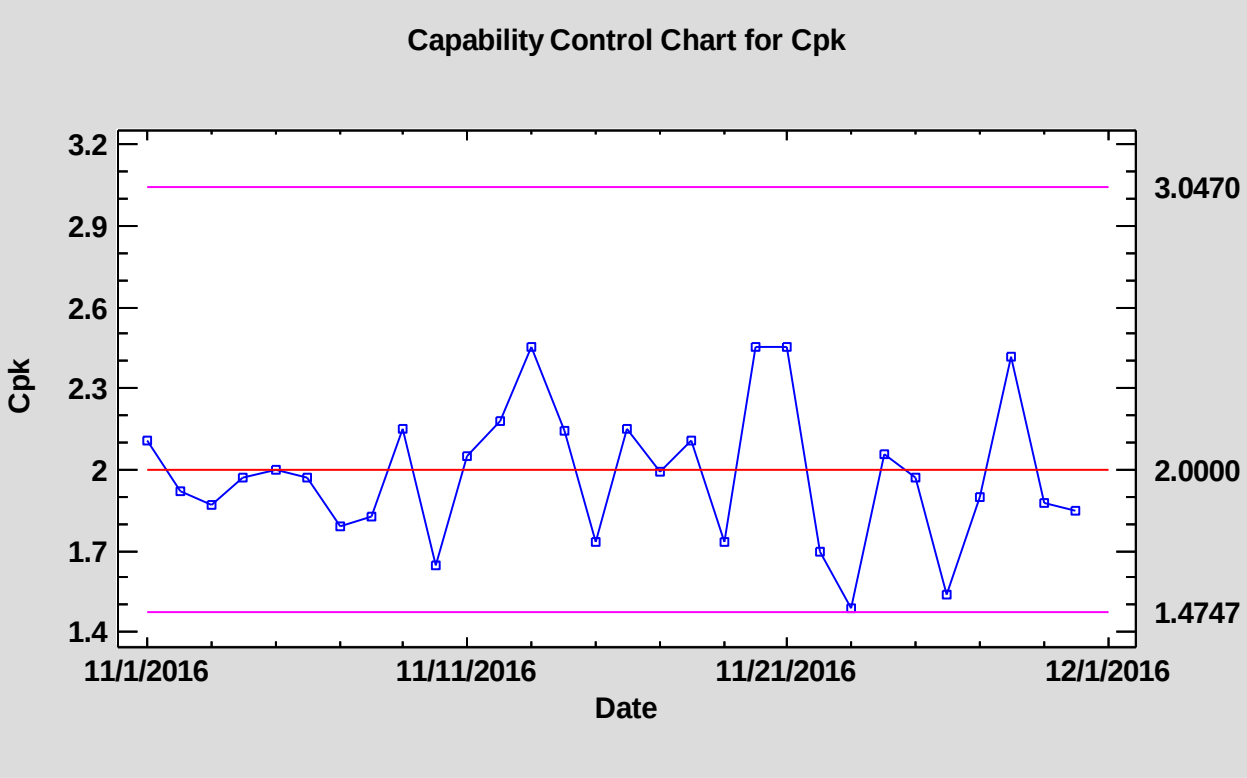
$$LCL = C_{pk,0}\left[1 - Z_{\frac{\alpha}{2}}\sqrt{\frac{1}{9m\bar{n}\,LCL^2} + \frac{1}{2\nu}}\right] \quad (4)$$

A similar calcution is used for P_{pk}.

The table shows the location of the control limits and the probability "Alpha" that any single plotted point will be beyond the control limit if the true index equals the target index.

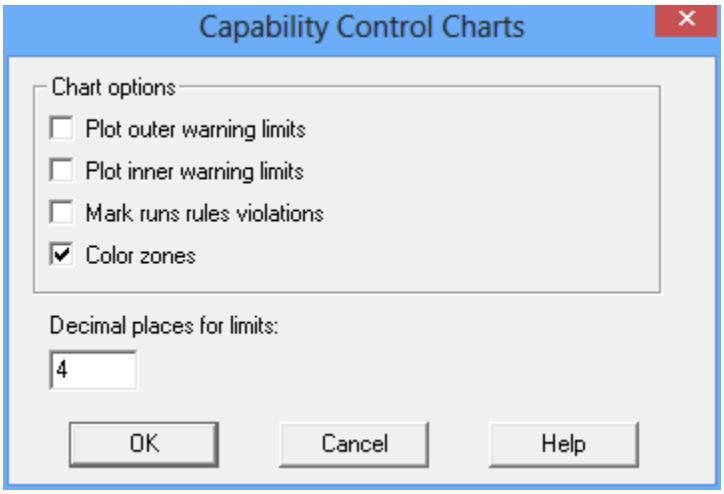
Control Chart

The *Control Chart* shows the plotted points with the centerline and control limit(s):



By default, points beyond the limits are flagged in red.

Pane Options



- **Plot outer warning limits:** if checked, lines are plotted 2/3rds of the way between the centerline and the control limits.

- **Plot inner warning limits:** if checked, lines are plotted 1/3rd of the way between the centerline and the control limits.

- **Mark runs rules violations:** if checked, points are also flagged if they violate specific runs rules (see below).

- **Color zones:** color zones green, yellow and red, corresponding the the control and warning limits.

- **Decimal places for limits:** the number of decimal places used to display the location of the centerline and control limits.

An example of a chart with colored zones is shown below.

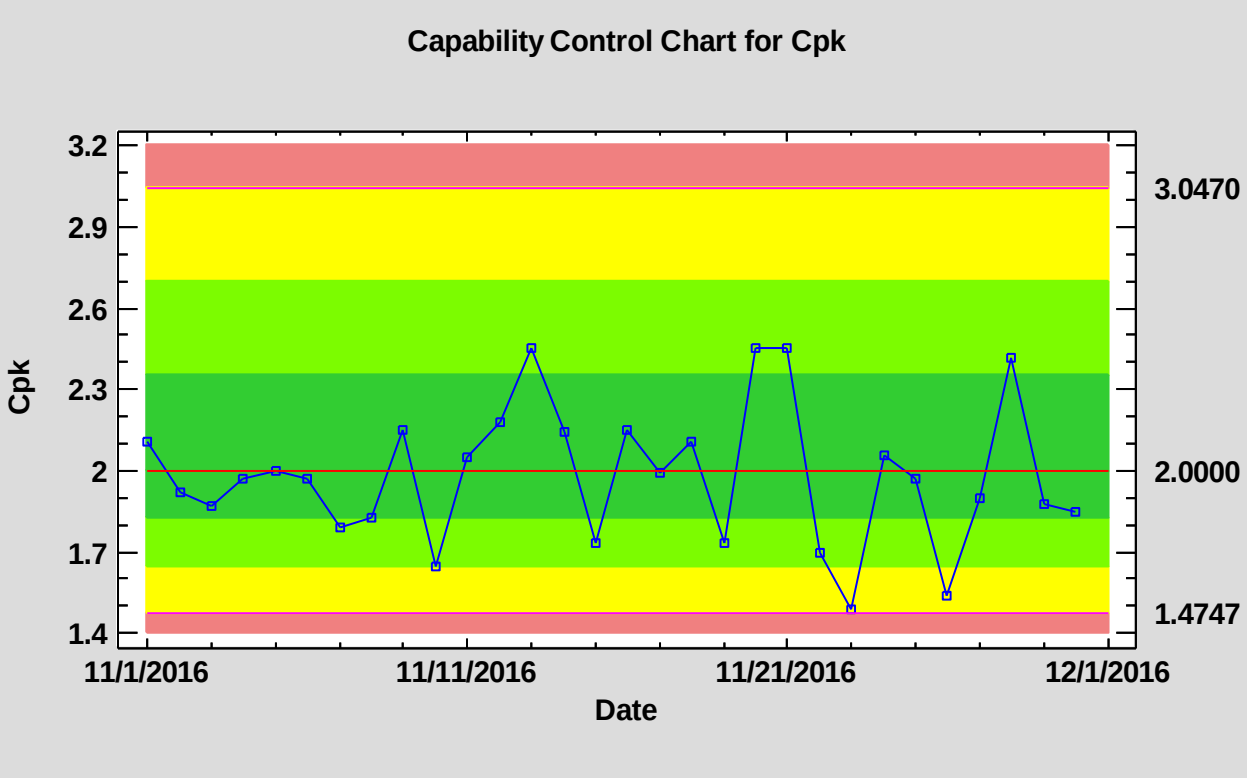


Chart Report

This pane tabulates the values plotted on the control chart:

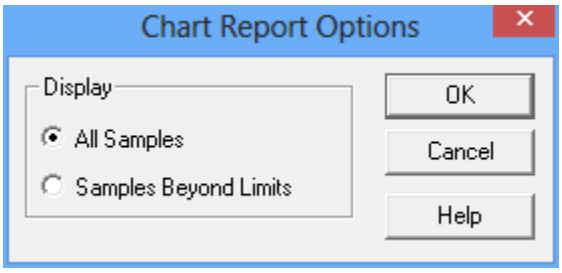
Chart Report

*** = Beyond Limits**

Sample	*Cpk*
11/1/2016	2.11
11/2/2016	1.92
11/3/2016	1.87
11/4/2016	1.97
11/5/2016	2.0
11/6/2016	1.97
11/7/2016	1.79
11/8/2016	1.83
11/9/2016	2.15
11/10/2016	1.65
11/11/2016	2.05
11/12/2016	2.18
11/13/2016	2.45
11/14/2016	2.14
11/15/2016	1.73
11/16/2016	2.15
11/17/2016	1.99
11/18/2016	2.11
11/19/2016	1.73
11/20/2016	2.45
11/21/2016	2.45
11/22/2016	1.7
11/23/2016	1.49
11/24/2016	2.06
11/25/2016	1.97
11/26/2016	1.54
11/27/2016	1.9
11/28/2016	2.42
11/29/2016	1.88
11/30/2016	1.85

Out-of-control points are indicated by an asterisk.

Pane Options



- **Display:** specify the samples to display in the report.

Runs Tests

The *Runs Tests* pane displays the results of standard tests applied to the control chart to look for unusual sequences of points.

Runs Tests
Rules
(A) runs above or below centerline of length 7 or greater.
(B) runs up or down of length 7 or greater.
(C) sets of 5 samples with at least 4 beyond inner warning limits
(D) sets of 3 samples with at least 2 beyond outer warning limits

Violations

Sample	*Capability Chart*

Depending on the default settings on the *Runs Tests* tab of the *Preferences* dialog box, STATGRAPHICS will look for up to 7 different types of patterns:

A. A group of 8 or more points, all above or all below the centerline.

B. A group of 8 or more points, all increasing or all decreasing.

C. A group of 5 points in which at least 4 are more than 1-sigma away from the centerline, on the same side of the centerline.

D. A group of 3 points in which at least 2 are more than 2-sigma away from the centerline, on the same side of the centerline.

E. A group of 15 or more points, all within 1-sigma.

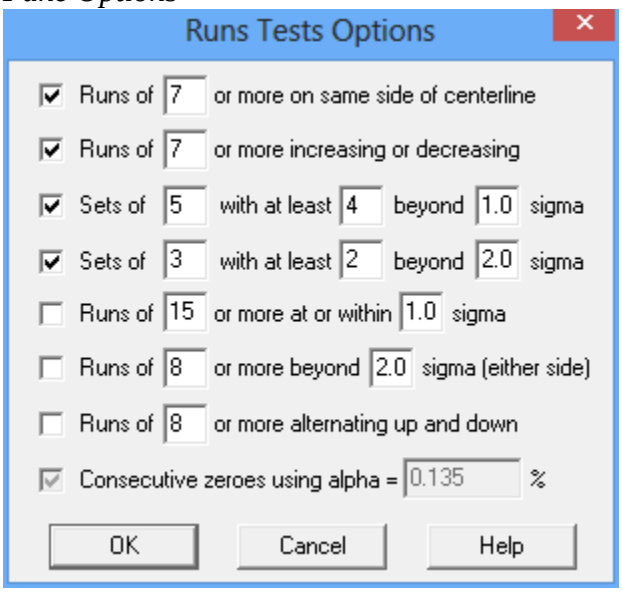
F. A group of 8 or more points, all beyond 2-sigma, but not necessarily on the same side of the centerline.

G. A group of 8 or more points, all following an alternating up and down pattern.

Any such runs will be indicated in the above table and also on the control charts (unless suppressed). NOTE: the rules are expressed above in terms of "1-sigma" and "2-sigma" to follow standard convention, with the control limits located at "3-sigma". In a capability control chart, "1-sigma" refers to 1/3 of the distance between the centerline and the control limit while "2-sigma" refers to 2/3rds of that distance.

Runs tests are designed to make standard Shewhart charts more sensitive to small shifts in the process.

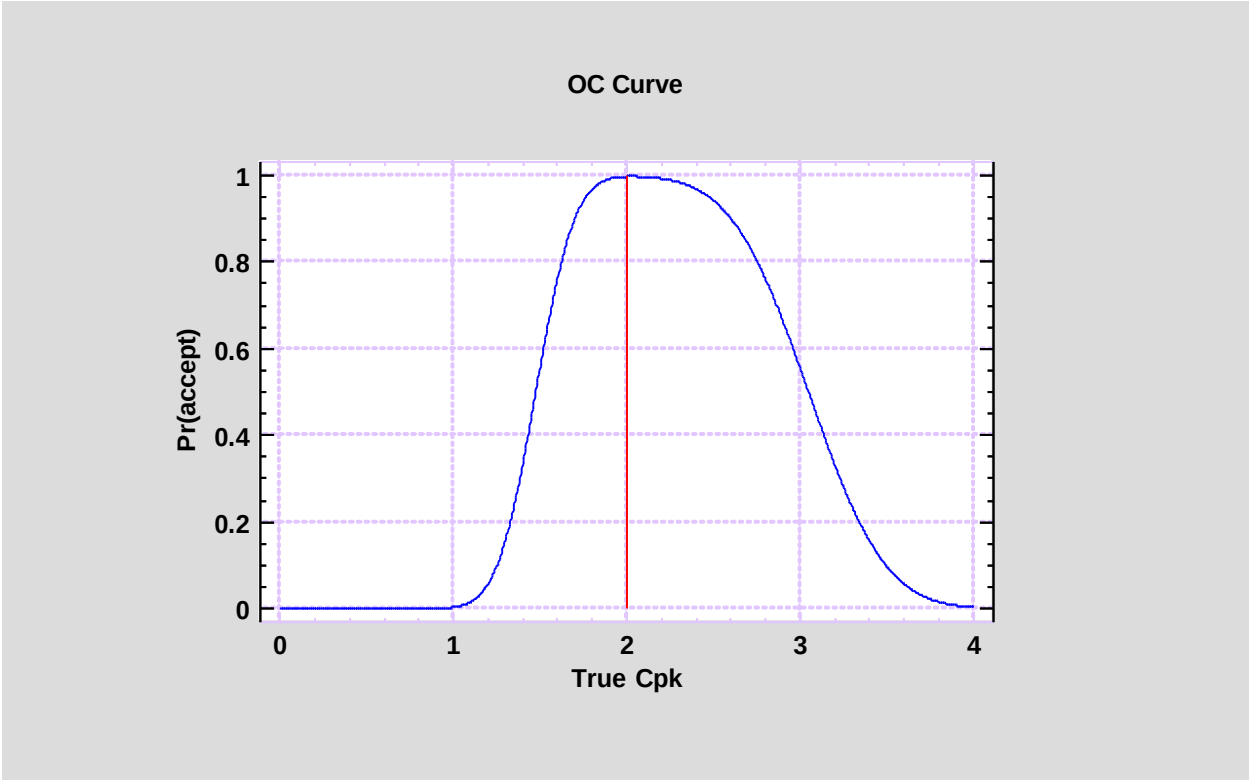
Pane Options



Select the runs tests to be applied and the parameters that define those tests. For example, some practitioners prefer to test for runs of length 8 rather than 7.

OC Curve

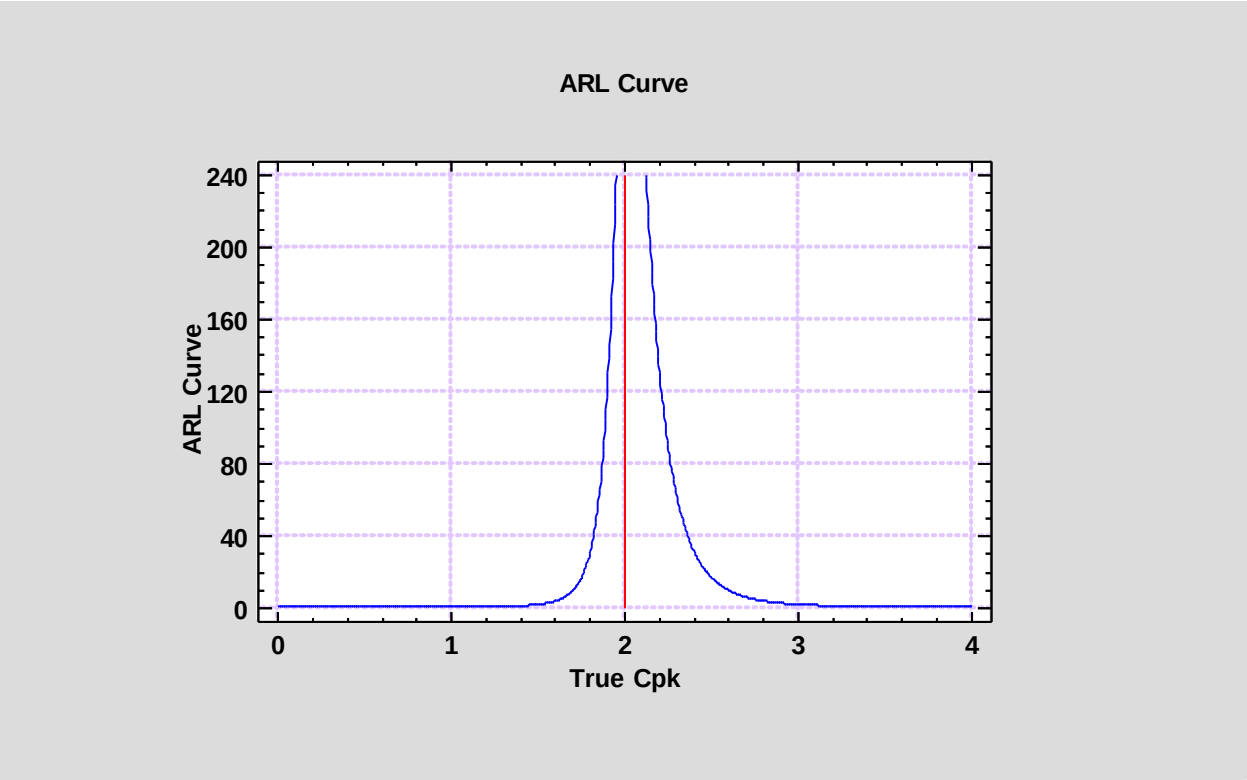
The *OC (Operating Characteristic) Curve* is designed to illustrate the properties of a Phase 2 control chart.



The chart displays the probability that a sample capability index will be within the control limits on the control chart, as a function of the true process index. For example, if the true process index were to shift from 2.0 to 1.5, the sample index would remain within the limits approximately 89% of the time and thus not generate an out-of-control signal. That probability drops to about 2% if the index shifts to 1.0.

ARL Curve

The *ARL Curve* is another way to view the performance of a Phase 2 control chart.



The ARL curve plots the average run length (average number of samples plotted up to and including the first point beyond the control limits) as a function of the true process capability index. Assuming that the index suddenly shifts to a new value, the chart shows how long it takes on average until an out-of-control signal is generated (not counting any signals from run rules violations). For very small shifts, it can take hundreds of samples on average to detect the shift.