# statgraphics 18®

## Average and Range Method
## (Gage Studies – Variables)

# statgraphics 18®
## centurion

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## Summary

The *Average and Range Method* estimates the repeatability and reproducibility of a measurement system based on a study in which *m* appraisers measure *n* items *r* times. It also estimates important quantities such as the total variation, the precision-to-tolerance ratio, the standard deviation of the measurement error, and the percent of study contribution from various error components. The *ANOVA Method* performs a similar analysis using a somewhat different statistical approach.

**Sample StatFolio:** *gageavgrange.sgp*

## Sample Data

The file *gage1.sgd* contains data from a typical variables gage study, taken from the third edition of the Automotive Industry Action Group's (AIAG) reference manual on <u>Measurement Systems Analysis, MSA</u> (2002). A partial list of the data in that file is shown below:

| Appraiser | Part | Trial | Measurement |
|-----------|------|-------|-------------|
| A | 1 | 1 | 0.29 |
| A | 2 | 1 | -0.56 |
| A | 3 | 1 | 1.34 |
| A | 4 | 1 | 0.47 |
| A | 5 | 1 | -0.80 |
| A | 6 | 1 | 0.02 |
| A | 7 | 1 | 0.59 |
| A | 8 | 1 | -0.31 |
| A | 9 | 1 | 2.26 |
| A | 10 | 1 | -1.36 |
| A | 1 | 2 | 0.41 |
| A | 2 | 2 | -0.68 |
| A | 3 | 2 | 1.17 |
| A | 4 | 2 | 0.50 |
| A | 5 | 2 | -0.92 |
| A | 6 | 2 | -0.11 |
| A | 7 | 2 | 0.75 |
| A | 8 | 2 | -0.20 |
| A | 9 | 2 | 1.99 |
| A | 10 | 2 | -1.25 |
| B | 1 | 1 | 0.08 |
| B | 2 | 1 | -0.47 |

The file contains a total of 90 rows, one for each of $r = 3$ measurements made by each of $m = 3$ operators on $n = 10$ parts.

**Note: Data reprinted from the Measurement Systems Analysis (MSA) Manual with permission of DaimlerChrysler, Ford and GM Supplier Quality Requirements Task Force.**

## Data Input

The first dialog box displayed by this procedure is used to indicate the structure of the data to be analyzed.
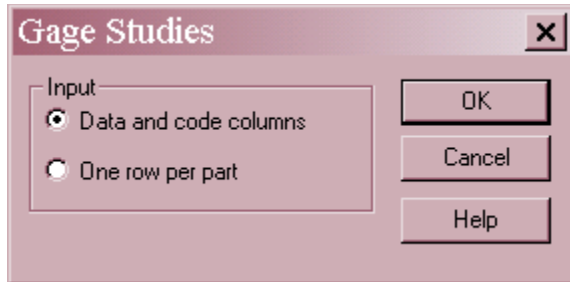


**Input:** The datasheet may be organized into either of two formats:

- *Data and Code Columns*: indicates that the datasheet contains a single column holding all the measurements. In this format, additional columns must be provided to identify which measurements correspond to which part and which appraiser. This is the type of data structure illustrated above.
- *One Row for Each Part*: indicates that the datasheet contains a single row for all measurements on a specific part. In this format, the column names are used to identify which measurements were made by which appraiser. An example of this data structure is shown below:

| Part | A_1 | A_2 | A_3 | B_1 | B_2 | B_3 | C_1 | C_2 | C_3 |
|------|------|------|------|------|------|------|------|------|------|
| 1 | 0.29 | 0.41 | 0.64 | 0.08 | 0.25 | 0.07 | 0.04 | -0.11 | -0.15 |
| 2 | -0.56 | -0.68 | -0.58 | -0.47 | -1.22 | -0.68 | -1.38 | -1.13 | -0.96 |
| 3 | 1.34 | 1.17 | 1.27 | 1.19 | 0.94 | 1.34 | 0.88 | 1.09 | 0.67 |
| 4 | 0.47 | 0.50 | 0.64 | 0.01 | 1.03 | 0.20 | 0.14 | 0.20 | 0.11 |
| 5 | -0.80 | -0.92 | -0.84 | -0.56 | -1.20 | -1.28 | -1.46 | -1.07 | -1.45 |
| 6 | 0.02 | -0.11 | -0.21 | -0.20 | 0.22 | 0.06 | -0.29 | -0.67 | -0.49 |
| 7 | 0.59 | 0.75 | 0.66 | 0.47 | 0.55 | 0.83 | 0.02 | 0.01 | 0.21 |
| 8 | -0.31 | -0.20 | 0.17 | -.63 | 0.08 | -0.34 | -0.46 | -0.56 | -0.49 |
| 9 | 2.26 | 1.99 | 2.01 | 1.80 | 2.12 | 2.19 | 1.77 | 1.45 | 1.87 |
| 10 | -1.36 | -1.25 | -1.31 | -1.68 | -1.62 | -1.50 | -1.49 | -1.77 | -2.16 |

If the data will be analyzed by other STATGRAPHICS procedures, *Data and Code Columns* is the preferred format since it follows the structure expected by most other procedures.

The second dialog box displayed depends on the setting in the first dialog box.

Data and Code Columns
If you select *Data and Code Columns* on the first dialog box, the second dialog box requests the name of the column containing the measurements and the columns containing appraiser and part indicators.
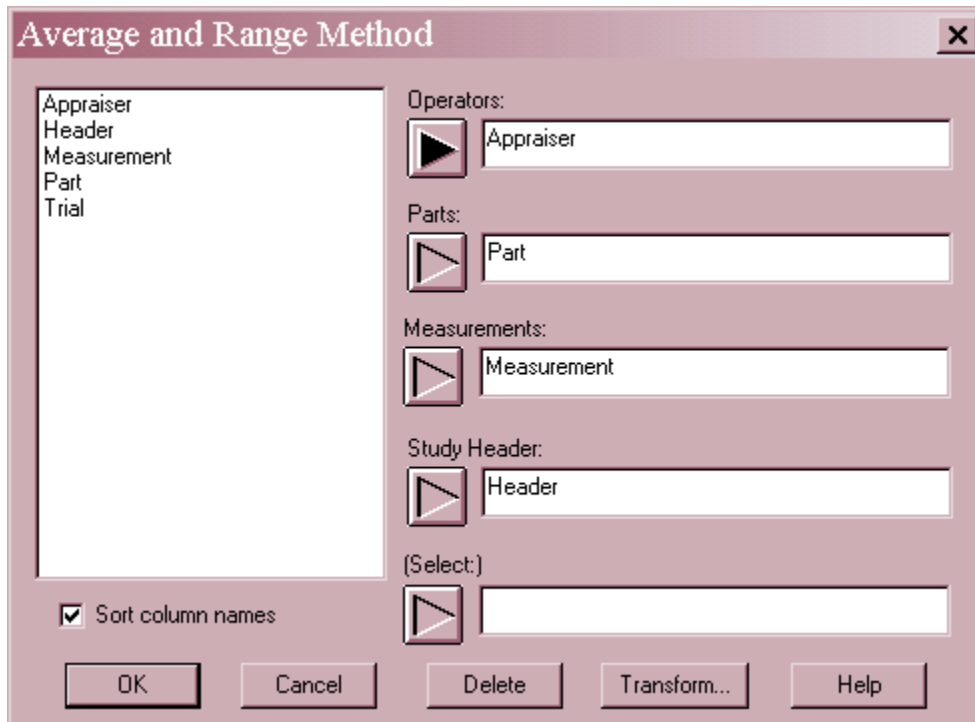


- **Operators**: numeric or non-numeric column indicating the appraiser corresponding to the measurements in each row.
- **Parts**: numeric or non-numeric column indicating the item corresponding to the measurements in each row.
- **Measurements**: numeric column containing the measurements.
- **Study Header**: optional header to be printed at the top of each output table.
- **Select**: subset selection.

If the study has *m* appraisers, *n* items, and *r* trials, there must be exactly *mnr* rows with non-missing data. Each operator-part combination must also have exactly *r* measurements (i.e., the study must be balanced).

One Row for Each Part

If you select *One Row for Each Part* on the first dialog box, the second dialog box requests the names of the columns containing the measurements and the number of appraisers.
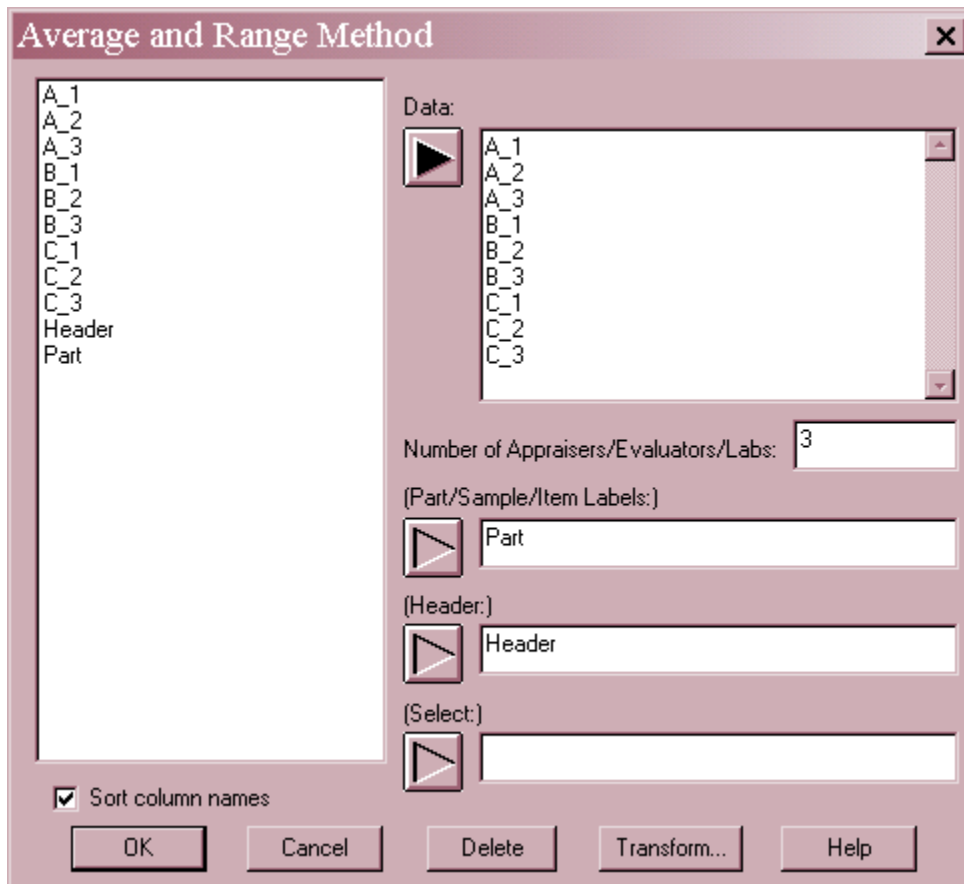


- **Data**: numeric columns containing the measurements. Each group of *m* columns is assumed to correspond to the same appraiser.
- **Number of Appraisers/Evaluators/Labs**: *m*, the number of appraisers. This number must be between 2 and 18 and divide evenly into the number of data columns.
- **Part/Sample/Item Labels**: optional labels for each item in the study. In no entry is made, the items will be numbered from 1 to *n*.
- **Study Header**: optional header to be printed at the top of each output table.
- **Select**: subset selection.

## Run Chart

When analyzing data from a gage study, a useful plot to examine first is the *Run Chart*.
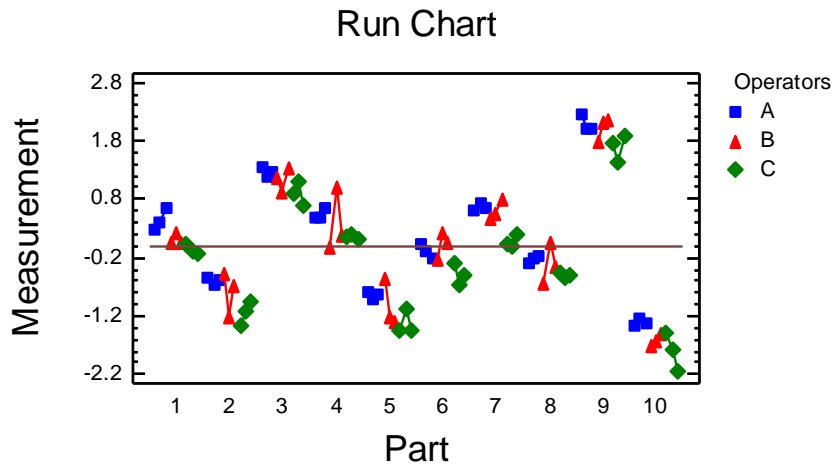


Run Chart

The *Run Chart* plots each of the measurements in the study, grouped by appraiser and part. If the measurement system is capable of distinguishing one part from another, the measurements should not be randomly scattered, but should show obvious grouping by parts. This is true in the sample data above, although you can also seen some differences amongst the operators.

## Operator and Part Plot

In estimating repeatability and reproducibility, the first step is to calculate statistics for each combination of operator and part:
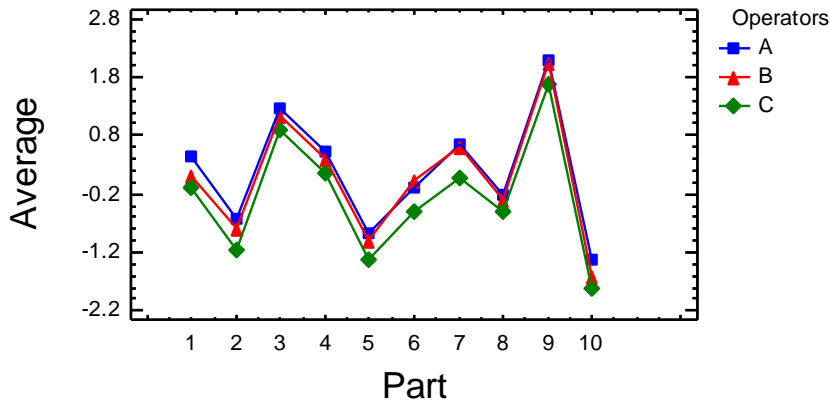
$\bar{x}_{ij}$ = average measurement made by operator *i* on part *j*

$R_{ij}$ = range of measurements made by operator *i* on part *j*

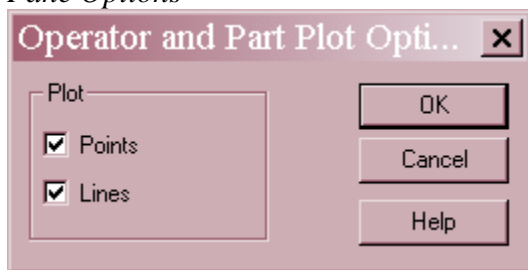$\bar{x}_i$ = average measurement made by operator *i*

The *Operator and Part* plot displays $\bar{x}_{ij}$, the operator by part averages:

## Gage Measurements by Appraiser



This plot is useful for showing any consistent differences between the appraisers. For example, Appraiser C appears to be consistently lower than Appraiser A.
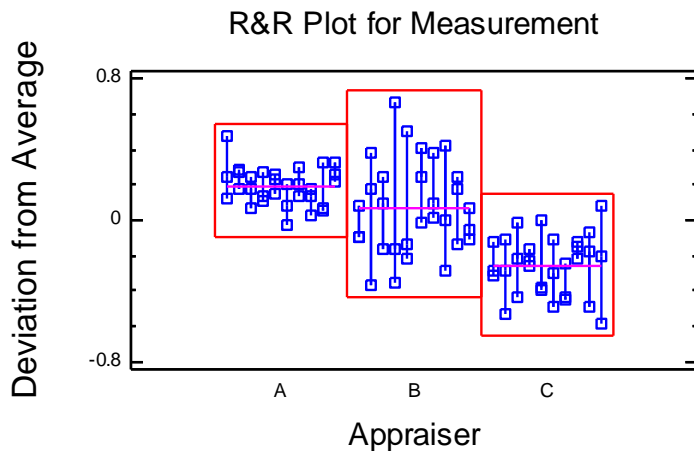
*Pane Options*



- **Points**: plot point symbols.
- **Lines**: connect the points with a line.

## R&R Plot

Another useful plot is the *R&R Plot*, which contains a single point for each measurement in the study:

### R&R Plot for Measurement

The vertical axis is scaled to show the difference between each measurement and the overall mean of all the measurements. The points are grouped by appraiser, and a horizontal line is drawn at the average measurement $\bar{x}_i$ for each appraiser. Vertical lines connect measurements made by the same appraiser on the same item.

It can be seen from the above plot that Appraiser C's measurements are smaller on average than those of the other two appraisers. On the other hand, Appraiser B shows considerably more variability. In fact, the repeatability of Appraiser B is poor, since there are several large discrepancies between repeated measurements on the same item.

## Range Charts

The operator by part ranges, $R_{ij}$, are displayed on two plots: the *Range Chart by Operator* and the *Range Chart by Plot*. The *Range Chart by Operator* may be used to determine whether there are any instances of usual discrepancies between repeated measurements on the same item.
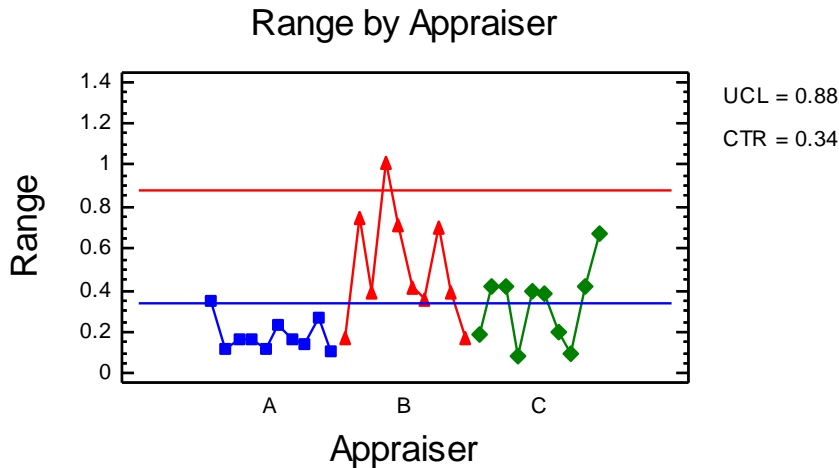
### Range by Appraiser



Included on the chart is a 3-sigma upper control limit. Any ranges beyond this limit, of which Appraiser B has one, indicate unusual events. In fact, Appraiser B shows consistently larger ranges than the other two appraisers.

The *Range Chart by Part* is used to detect items which are unusually difficult to measure.

### Range by Part
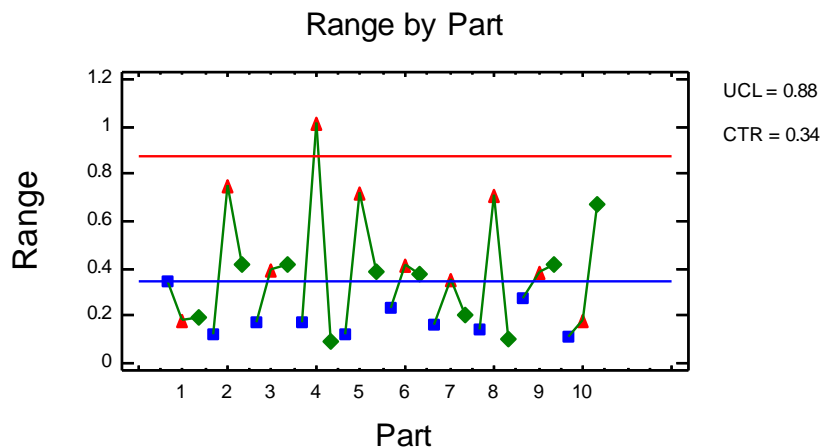


In this case, no single part seems to have a large range for more than one appraiser.

*Pane Options*
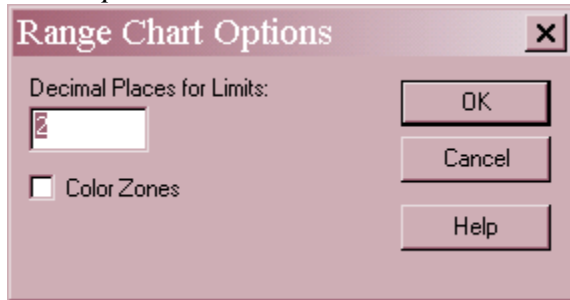


- **Decimal Places for Limits:** number of decimal places to be displayed for the control limits.

- **Color Zones:** check this box to display green, yellow and red zones.

## Analysis Summary

Having examined the data, it is then time to view the numerical results. The *Analysis Summary* displays estimates of the variability due to repeatability, reproducibility, and parts.

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**Gage R&R - Average and Range Method - Measurement**

AIAG Example
Operators: Appraiser
Parts: Part
Measurements: Measurement

3 operators  10 parts  3 trials

Average range = 0.341667     Range of x-bars = 0.444667

| Measurement Unit | Estimated Sigma | Percent Total Variation | Estimated Variance | Percent Contribution | Percent of R&R |
|---|---|---|---|---|---|
| Repeatability | 0.200894 | 17.5269 | 0.0403582 | 3.07193 | 43.34 |
| Reproducibility | 0.229711 | 20.0411 | 0.0527673 | 4.01646 | 56.66 |
| R & R | 0.305165 | 26.624 | 0.0931255 | 7.08839 | 100.00 |
| Parts | 1.10483 | 96.3907 | 1.22065 | 92.9116 | |
| Total Variation | 1.1462 | 100.0 | 1.31378 | | |

Number of distinct categories (ndc): 5

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This summary displays:

- **Average range** – the average range over all combinations of operators and parts.

- **Range of x-bars** – the range of the average measurements $\bar{x}_i$ amongst the *m* operators.

- **Repeatability** - estimate of the variation between measurements made by the same appraiser on the same part, usually attributed to the instrument.

- **Reproducibility** - estimate of the variation between measurements made by different appraisers on the same part, usually attributed to the appraiser.

- **R & R** - estimate of the total measurement error, calculated by adding the variances due to repeatability and reproducibility.

- **Parts** – estimate of the actual variability among the items measured. If the measurement process is capable of separating good items from bad items, this should be large compared to the variability of the measurement process.

- **Total** – sum of the variability due to the measurement process and the actual variability amongst the items.

For each measurement unit (component), the columns of the table show:

- **Estimated Sigma** – the estimated standard deviation $\hat{\sigma}_{componen}$.

- **Percent Total Variation** – the percentage of the total standard deviation:

$$100\frac{\hat{\sigma}_{component}}{\hat{\sigma}_{total}}\%  \tag{1}$$

  where

$$\hat{\sigma}_{total} = \sqrt{\hat{\sigma}_{repeat}^2 + \hat{\sigma}_{repro}^2 + \hat{\sigma}_{parts}^2}  \tag{2}$$

- **Estimated Variance** - the estimated variance of each component $\hat{\sigma}_{componen}^2$.

- **Percent Contribution** – the percentage of the total variance:

$$100\frac{\hat{\sigma}_{component}^2}{\hat{\sigma}_{total}^2}\%  \tag{3}$$

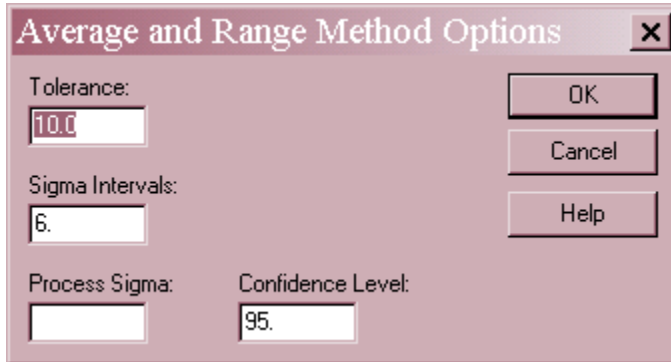- **Percent of R&R** – the percentage of the overall measurement variance

$$100\frac{\hat{\sigma}_{repeat}^2}{\hat{\sigma}_{R\&R}^2}\% \quad \text{and} \quad 100\frac{\hat{\sigma}_{repro}^2}{\hat{\sigma}_{R\&R}^2}\%  \tag{4}$$

Of particular interest is $\hat{\sigma}_{total}$, often labeled *TV* for total variation. The rule of thumb cited by the AIAG is that if *TV* is less than 10%, then the measurement system is usually deemed to be acceptable. In certain cases, values between 10% and 30% may also be acceptable, depending on the circumstances. The relative percentages of repeatability and reproducibility can also be helpful in isolating the largest source of variability in the measurement process.

One more statistic is also displayed:

- **Number of distinct categories (ndc)** – According to the AIAG (2002), *ndc* represents "the number of distinct categories that can be reliably distinguished by the measurement system." It is basically a measure of how many 97% confidence intervals for the true value being measured can fit within the range of expected part-to-part variation. Values greater than or equal to 5 are desirable.

## Analysis Options



- **Tolerance**:  the distance between the specification limits, USL - LSL. The value entered in this field is used by the *Tolerance Analysis,* described below, to compute a *precision-to-tolerance* (P/T) ratio.

- **Sigma Intervals**: the sigma multiple *K* used to compare the spread of the measurement error relative to the distance between the specifications. The value 6.0 gives 99.73% coverage based upon a normal distribution. For 99% coverage, change the value to *K = 5.15*. The default value of *K* is determined by the settings on the *Gage Studies* tab of the *Preferences* selection on the *Edit* menu.

- **Process Sigma**: the value of the process standard deviation, if known. If a value is supplied, the estimated total variation $\hat{\sigma}_{total}$ will be replaced by this value and the part-to-part variation updated accordingly. This impacts the estimated percentage contributions of all components.

- **Confidence Level**: percentage used in calculating confidence intervals.

## Tolerance Analysis

An alternative method for assessing the acceptability of a measurement system is to compare the estimated measurement variation to the distance covered by the specification limits for the variable being measured. If LSL and USL represent the lower specification limit and upper specification limit, respectively, then the tolerance is defined as

$$tolerance = USL - LSL \tag{5}$$

The *Tolerance Analysis* pane compares the estimates of the various measurement unit variations to the tolerance.

| Tolerance Analysis |
| --- |
| AIAG Example |
| 3 operators  10 parts  3 trials |

Tolerance = 10.0

| Measurement Unit | 6.0 Std. Dev. | Percent of Tolerance |
|---|---|---|
| Repeatability | 1.20536 | 12.0536 |
| Reproducibility | 1.37827 | 13.7827 |
| R & R | 1.83099 | 18.3099 |
| parts | 6.62898 | 66.2898 |

The table shows:

- **6.0 Std. Dev.** - displays $K\hat{\sigma}$ for each of the various error components. If $K$ equals 6.0, this estimates the interval within which the associated error component will lie 99.73% of the time.

- **Percent of tolerance** – the percentage of the tolerance represented by $K\hat{\sigma}$:

$$100\frac{K\hat{\sigma}_{component}}{tolerance}\%$$ (6)

Of particular interest is the *Percent of Tolerance* due to *R&R*, often called the "precision- to-tolerance ratio"or P/T. Basically, P/T is a measure of how wide the measurement error distribution is compared to the specifications for the item being measured. Values of P/T less than 10% usually imply an acceptably small measurement error, although P/T may be as high as 30% in some cases and still be acceptable.

## Confidence Intervals

Since the percentage variations for each component are estimates from a limited amount of data, they are almost certain to deviate somewhat from their process values. The *Confidence Intervals* pane displays the amount of uncertainty surrounding those estimates.

**Confidence Intervals**
AIAG Example
3 operators  10 parts  3 trials

**95.0 Confidence Intervals**

| | Lower Limit | 6.0 Std. dev. | Upper Limit |
|---|---|---|---|
| Repeatability | 1.01581 | 1.20536 | 1.48255 |
| Reproducibility | 0.718204 | 1.37941 | 8.66925 |
| R & R | 1.54808 | 1.83185 | 2.24398 |
| parts | 4.45045 | 6.62898 | 12.9145 |

Displayed in the table are the estimates $K\hat{\sigma}$, together with their upper and lower confidence limits. In the example, the estimated standard deviation for R&R equals 0.305, so the 6-sigma spread for the measurement error distribution is approximately 1.83. With 95% confidence, however, one can state only that the spread is somewhere between 1.55 and 2.24. This equates to a range of uncertainty in the P/T ratio of approximately 15.5% to 22.4%.

## Box and Whisker Plot

The box-and-whisker plot provides an additional comparison between the appraisers.

### Box-and-Whisker Plot
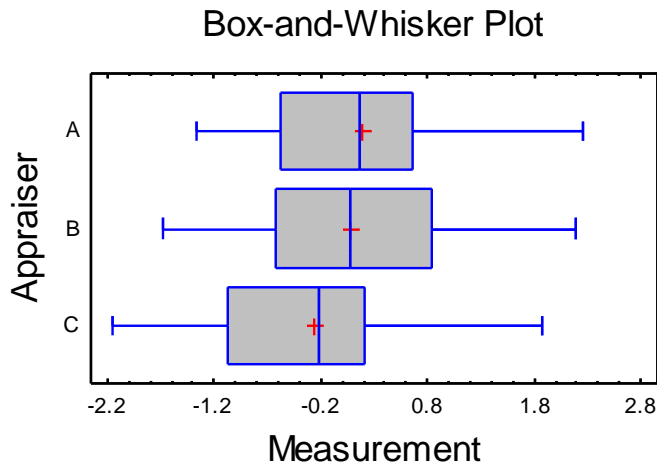


For each appraiser, a box-and-whisker plot is drawn as follows:

- The rectangular *box* covers the central 50% of an appraiser's measurements, ranging between the lower quartile and the upper quartile.

- A vertical line is drawn within the box at the *median* for that appraiser.

- A plus sign is drawn to indicate the *mean* measurement $\bar{x}_i$ of each appraiser.

- *Whiskers* are drawn from each end of the box to the minimum and maximum value for each operator, unless outside points are detected, in which case the whiskers are drawn to the most extreme data values that are not outside points.

- Any *outside points* are indicated using point symbols such as a small square, or a square with a plus sign through it if the points are *far outside*.

For more details on outside points and other features of box-and-whisker plots, refer to the documentation for the standalone *Box-and-Whisker Plot* procedure.

In the above plot, no single measurements appear to be outliers since there are not any outside points.

*Pane Options*
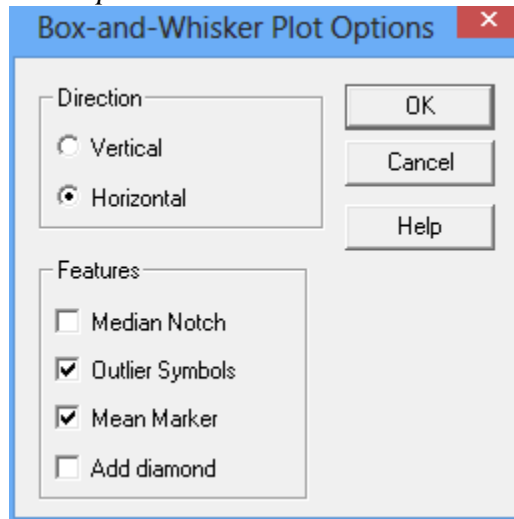


- **Direction**: the orientation of the plot, corresponding to the direction of the whiskers.

- **Median Notch**: if selected, a notch will be added to the plot showing an approximate 100(1-α)% confidence interval for the median at the default system confidence level (set on the *General* tab of the *Preferences* dialog box on the *Edit* menu).

- **Outlier Symbols**: if selected, indicates the location of outside points.

- **Mean Marker**: if selected, shows the location of the sample mean as well as the median.

- **Add diamond:** if selected, a diamond will be added to the plot showing a 100(1-α)% confidence interval for the mean at the default system confidence level.

## Calculations

- **Average range**

$$\bar{R} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} R_{ij}}{mn} \tag{7}$$

- **Range of x-bars**

$$\bar{R}_{\bar{x}} = Range(\bar{x}_1, \bar{x}_2, ..., \bar{x}_m) \tag{8}$$

- **Repeatability**

$$\hat{\sigma}_{repeat} = \frac{\bar{R}}{d_2^*(r,mn)} \tag{9}$$

where

$$d_2^*(s,t) = \left(1 + \frac{1}{4v}\right)d_2(s) \tag{10}$$

and

$$v = \frac{1}{-2 + 2\sqrt{1 + \frac{2[d_3(s)/d_2(s)]^2}{t}}} \tag{11}$$

- **Reproducibility**

$$\hat{\sigma}_{repro} = \sqrt{\left(\frac{\bar{\bar{R}}_{\bar{x}}}{d_2^*(m,1)}\right)^2 - \frac{\hat{\sigma}_{repeat}}{mn}} \tag{12}$$

- **Combined variability from R & R**

$$\hat{\sigma}_{R\&R} = \sqrt{\hat{\sigma}_{repeat}^2 + \hat{\sigma}_{repro}^2} \tag{13}$$

- **Variability due to parts**

$$\hat{\sigma}_{parts} = \frac{\bar{R}_{\bar{x}_j}}{d_2^*(n,1)} \quad \text{if the process sigma is not specified} \tag{14}$$

$$\hat{\sigma}_{parts} = \sqrt{\sigma_{process}^2 - \hat{\sigma}_{R\&R}^2} \quad \text{if the process sigma is specified} \tag{15}$$

- **Total variability**

$$\hat{\sigma}_{Total} = \sqrt{\hat{\sigma}_{repeat}^2 + \hat{\sigma}_{repro}^2 + \sigma_{parts}^2} \tag{16}$$

Values for $d_2(s)$ and $d_3(s)$ are tabulated in textbooks such as <u>Introduction to Statistical Quality Control, sixth edition</u>, by Douglas Montgomery (Wiley, 2009).