Life Tables (Intervals)

Summary

The **Life Tables (Intervals)** procedure is designed to analyze data containing lifetimes or times until failure, where the value of each lifetime is known to fall within a specified interval. The data may include censoring, in which some failure times are not known exactly due to withdrawals of items from the test before they fail. The intervals are assumed to begin at 0 and be adjacent and nonoverlapping. Nonparametric estimates of the survival and hazard functions are obtained and plotted. Percentiles are also calculated.

If desired, the data for more than one group may be specified. In such cases, a separate estimate of the survival function for each group will be derived.

Sample StatFolio: *lifetable intervals.sgp*

Sample Data:

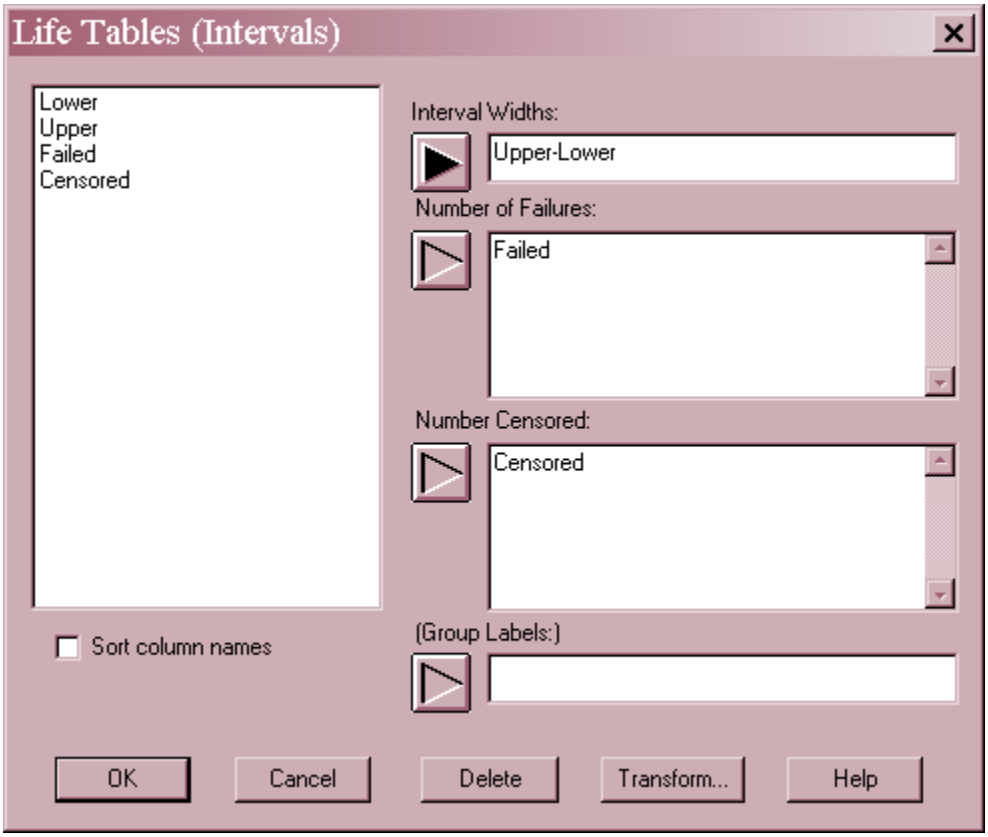
The file *life1.sgd* contains the data from a life test on $n = 68$ battery cells, reported by Meeker and Escobar (1998). Records were kept tabulating the number of cells that failed or were withdrawn from the test during each 50-hour period, as shown below:

Lower	*Upper*	*Failed*	*Withdrawn*
0	50	1	5
50	100	0	6
100	150	1	1
150	200	4	6
200	250	1	2
250	300	1	1
300	350	1	2
350	400	4	2
400	450	0	0
450	500	4	3
500	550	2	1
550	600	2	0
…	…	…	…
1650	1700	1	0

For example, 1 cell failed and 5 were withdrawn between 0 and 50 ampere-hours. 0 failed and 6 were withdrawn after more than 50 but less than or equal to 100 ampere-hours. The entire table contains 34 intervals up to and including 1700 ampere-hours. Note that the intervals are adjacent to each other, i.e., the lower limit of one interval is the upper limit of the interval preceding it.

Data Input

The data input dialog box requests information about each interval:



- **Interval Widths:** the widths of adjacent, non-overlapping intervals covering intervals of time in which the failure data has been tabulated. The first interval is assumed to start at time $t = 0$. Subtracting the lower endpoint from the upper endpoint is a simple way to calculate the width of the intervals.

- **Number of Failures:** one or more numeric columns containing the number of items that failed during each interval. If the data is divided into groups, enter a separate column for each group.

- **Number Censored:** one or more numeric columns containing the number of items that were withdrawn from the test during each interval. The number of columns entered must be the same as for *Number of Failures*.

- **Group Labels:** optional labels for each group of failure data.

Analysis Summary

The *Analysis Summary* displays a table showing the estimated survival, hazard, and density functions within each interval. Two sections of that table are shown below:

Life Tables (Intervals) - Upper-Lower
Interval Widths: Upper-Lower
Number of Failures: Failed
Number Censored: Censored

Life Table

Interval	*Number of Failures*	*Number Withdrawn*	*Number at Risk*	*Cumulative Survival*	*Hazard*	*Density*
0.0-50.0	1	5	65.5	1.000000	0.000308	0.000305
				(0.000000)	(0.000308)	(0.000303)
50.0-100.0	0	6	59.0	0.984733	0.000000	0.000000
				(0.015150)	(0.000000)	(0.000000)
100.0-150.0	1	1	55.5	0.984733	0.000364	0.000355
				(0.015150)	(0.000364)	(0.000352)
150.0-200.0	4	6	51.0	0.966990	0.001633	0.001517
				(0.023032)	(0.000816)	(0.000729)
200.0-250.0	1	2	43.0	0.891148	0.000471	0.000414
				(0.042140)	(0.000471)	(0.000410)
250.0-300.0	1	1	40.5	0.870423	0.000500	0.000430
				(0.045974)	(0.000500)	(0.000425)
300.0-350.0	1	2	38.0	0.848931	0.000533	0.000447
				(0.049609)	(0.000533)	(0.000442)
350.0-400.0	4	2	35.0	0.826591	0.002424	0.001889
				(0.053096)	(0.001210)	(0.000897)
400.0-450.0	0	0	30.0	0.732123	0.000000	0.000000
				(0.064712)	(0.000000)	(0.000000)
450.0-500.0	4	3	28.5	0.732123	0.003019	0.002055
				(0.064712)	(0.001505)	(0.000970)
500.0-550.0	2	1	22.5	0.629369	0.001860	0.001119
				(0.073238)	(0.001314)	(0.000766)
550.0-600.0	2	0	20.0	0.573425	0.002105	0.001147
				(0.076671)	(0.001487)	(0.000784)
600.0-650.0	1	0	18.0	0.516083	0.001143	0.000573
				(0.079001)	(0.001142)	(0.000564)
650.0-700.0	2	1	16.5	0.487412	0.002581	0.001182
				(0.079645)	(0.001821)	(0.000807)
700.0-750.0	1	0	14.0	0.428331	0.001481	0.000612
				(0.080203)	(0.001480)	(0.000601)
750.0-800.0	0	0	13.0	0.397736	0.000000	0.000000
				(0.080097)	(0.000000)	(0.000000)
800.0-850.0	3	0	13.0	0.397736	0.005217	0.001836
				(0.080097)	(0.002987)	(0.001000)
850.0-900.0	0	1	9.5	0.305951	0.000000	0.000000
				(0.077177)	(0.000000)	(0.000000)

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1500.0-1550.0	1	0	2.0	0.133854	0.013333	0.001339
				(0.077066)	(0.012571)	(0.001221)
1550.0-1600.0	0	0	1.0	0.066927	0.000000	0.000000
				(0.061028)	(0.000000)	(0.000000)
1600.0-1650.0	0	0	1.0	0.066927	0.000000	0.000000
				(0.061028)	(0.000000)	(0.000000)
1650.0-1700.0	1	0	1.0	0.066927	0.040000	0.001339
				(0.061028)	(0.000000)	(0.001221)
Total	33	35				

The columns in the table contain information about each of the intervals:

- **Interval** - the endpoints of the interval. Each interval is assumed to be closed at the upper end, i.e., the first interval corresponds to times for which $0 < t \le 50$. The width of each interval will be represented by w_i.

- **Number of Failures** - the number of units d_i failing within the interval. It is assumed that the failure times are equally likely to have occurred anywhere within the interval.

- **Number Withdrawn** - the number of units c_i removed from testing within the interval. It is assumed that the removal times are equally likely to have occurred anywhere within the interval.

- **Number at Risk** - the average number of items at risk during the interval, computed from

$$r_i = n - \sum_{j=1}^{i-1} d_j - \sum_{j=1}^{i-1} c_j - \frac{c_i}{2} \tag{1}$$

  This is equal to the number of units still operating at the start of the interval minus one-half of the number of items removed during the interval.

- **Cumulative Survival** - the estimated survival function at the start of the interval, given by

$$\hat{S}_i = \prod_{j=1}^{i-1} (1 - p_i) = \prod_{j=1}^{i-1} q_i \tag{2}$$

  where q_i is the proportion of items at risk during the interval which failed during that interval:

$$q_i = \frac{d_i}{r_i} \tag{3}$$

  The standard error of the survivor function, displayed in parentheses, is calculated using Greenwood's formula:

$$s\hat{e}.\left[\hat{S}_i\right] = \hat{S}_i \sqrt{\sum_{j=1}^{i-1} \frac{q_j}{r_j p_j}} \tag{4}$$

- **Hazard Function** - the estimated hazard function over the interval, calculated by

$$H_i = \frac{2p_i}{w_i(2 + p_i)} \tag{5}$$

  Its standard error is given by

$$s\hat{e}.\left[\hat{H}_i\right] = \hat{H}_i \sqrt{\frac{\left(1-\left(w_i \hat{H}_i / 2\right)^2\right)}{r_i q_i}} \qquad (6)$$

- **Density Function** - the nonparametric estimate of the failure time density function during the interval, given by

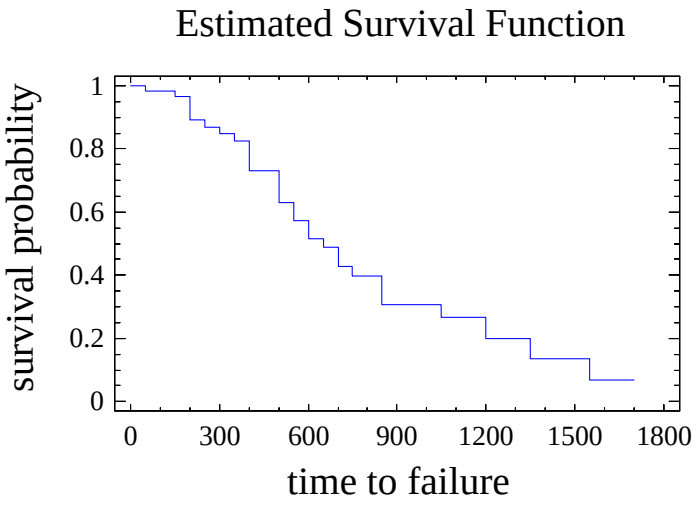
$$\hat{f}_i = \frac{S_i q_i}{w_i} \qquad (7)$$

with standard error equal to

$$s\hat{e}.\left[\hat{f}_i\right] = \frac{\hat{S}_i q_i}{w_i} \sqrt{\sum_{j=1}^{i-1} \frac{q_j}{r_j p_j} + \frac{p_i}{r_i q_i}} \qquad (8)$$

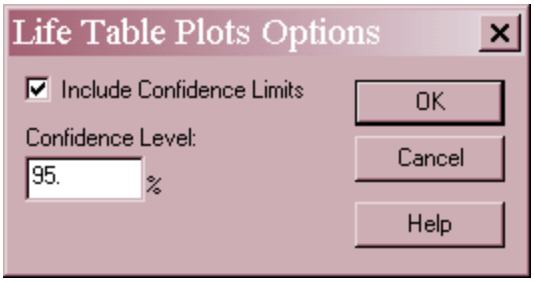
Survival Function

The *Survival Function* plots the estimated probability that an item will survive until time *t*:



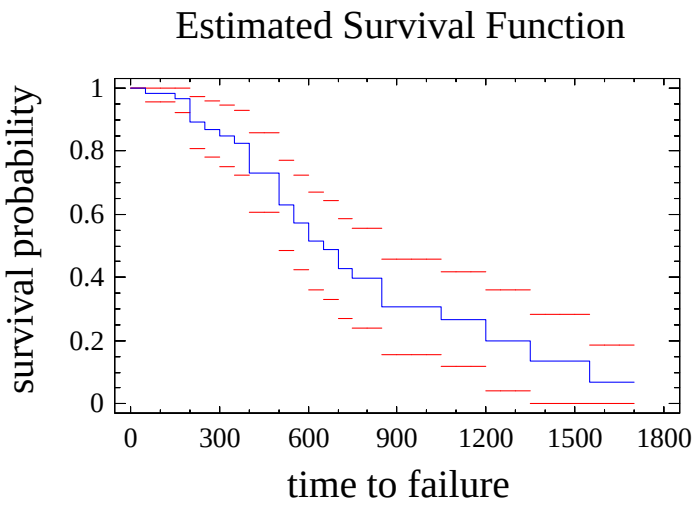
It decreases according to a step function, changing at the end of each interval in which at least 1 failure occurred.

Pane Options



- **Include confidence intervals**: if selected, confidence intervals will be added to the plot.
- **Confidence Level**: percentage of confidence for the intervals.

Example: Survival Function with 95% Confidence Intervals

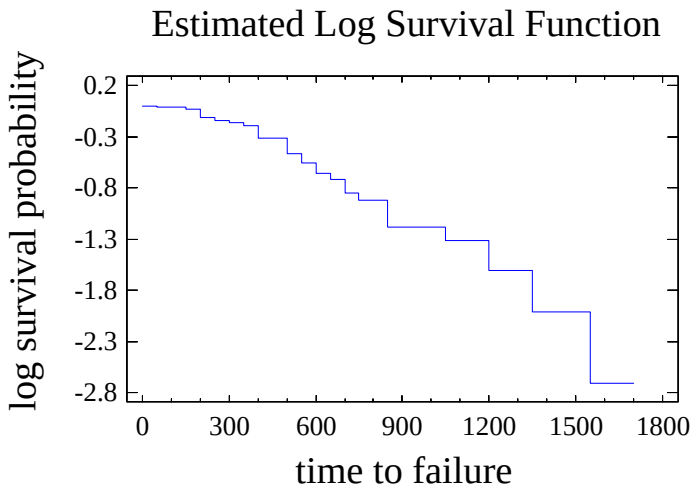


The confidence intervals are calculated from:

$$\hat{S}_i \pm z_{.025}\left[s\hat{e}.(\hat{S}_i)\right] \qquad (9)$$

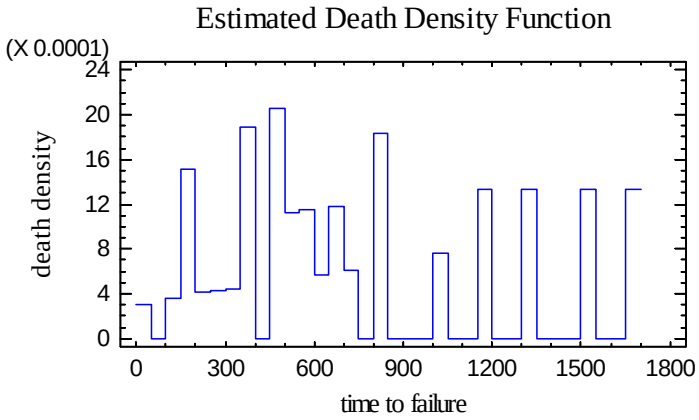
Log Survival Function

The *Log Survival Function* is the natural logarithm of the survival function:



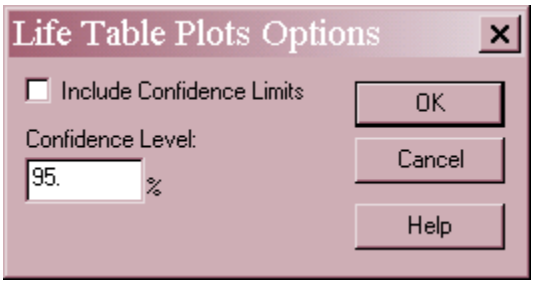
Death Density Function

The death density function shows a nonparametric estimate of the probability density function for time until failure:



For small data sets, it may not be very smooth.

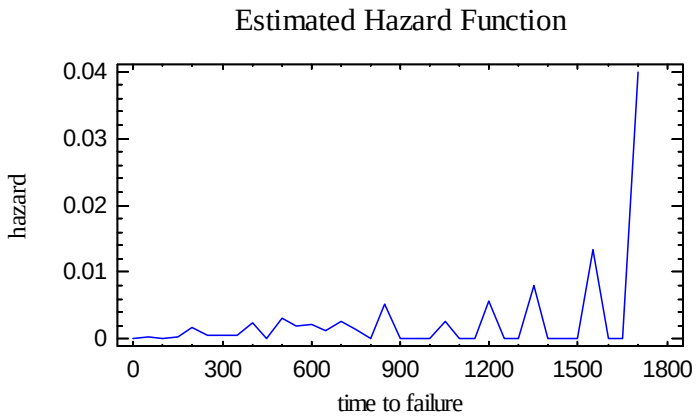
Pane Options



- **Include confidence intervals**: if selected, confidence intervals will be added to the plot.
- **Confidence Level**: percentage of confidence for the intervals.

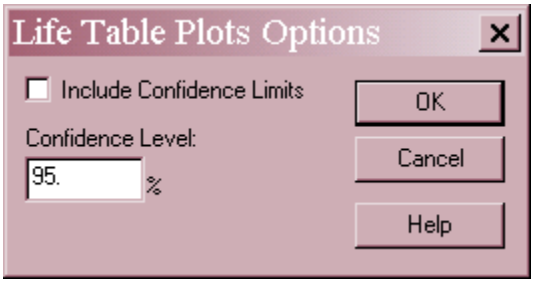
Hazard Function

The *Hazard Function* is an estimate of the instantaneous rate of failure:



The units of the hazard function is the fraction of items failed per unit time. In the plot above, the hazard function shows a general increase, particularly after 1000 ampere-hours, implying that the rate at which items are failing increases as they get older.

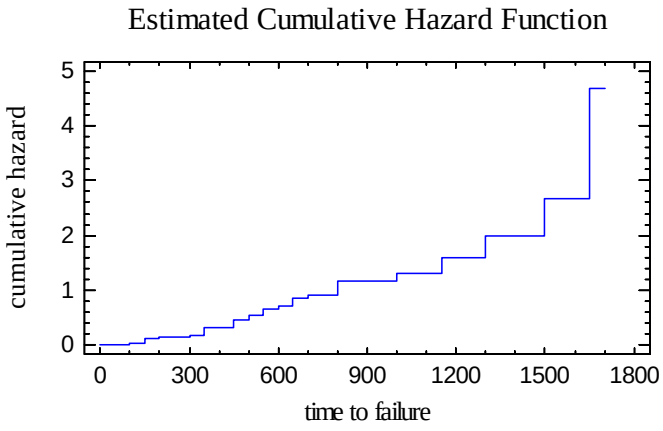
Pane Options



- **Include confidence intervals**: if selected, confidence intervals will be added to the plot.
- **Confidence Level**: percentage of confidence for the intervals.

Cumulative Hazard Function

The cumulative hazard function at *t* is the integral of the hazard function from 0 to *t*:



It can be used analytically to obtain the cdf or hazard function.

Percentiles

The *Percentiles* pane shows a table of estimated times at which given percentages of the item will still be operating:

Percentile Table

Percentile	*Estimate*	*Standard Error*
90.0	194.164	46.1576
75.0	390.538	44.7325
50.0	628.047	205.521
25.0	1163.23	186.771
10.0	1525.29	264.135

Percentiles are computed by finding the last interval *j* for which the estimated survivor function at the start of the interval is greater than or equal to the percentage desired, and then interpolating over that interval to find the desired percentile. The standard error of the estimated percentile is given by

$$\frac{1}{2\hat{f}_j\sqrt{r_j}} \qquad (10)$$

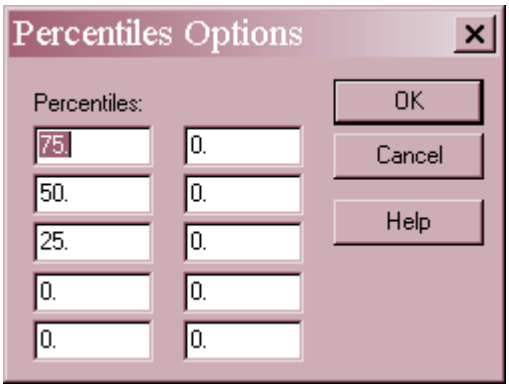
For example, the above table estimates that 10% of the cell batteries will still be operating after 1525 hours. An approximate 95% confidence interval is

$$1525 \pm 1.96(264)$$

or

1525 ± 517 ampere-hours.

Pane Options



- **Percentiles:** percentages at which to calculate the percentiles.

Group Comparisons

If more than one group of data has been entered, the above tables and plots will show separate estimates for each group. In addition, the *Group Comparisons* pane will summarize the data in each group:

Comparison of Groups

				Proportion
Group	*Total*	*Failed*	*Withdrawn*	*Withdrawn*
Group 1	68	33	35	0.5147
Total	68	33	35	0.5147

The table shows the total number of items in each group, the number in each group that failed, the number in each group that were withdrawn (censored), and the proportion of withdrawn items.

In the current example, there is only one group. Of the $n = 68$ items in that group, approximately 51.5% were withdrawn before they failed.

